Electoral Accountability in Multi-Member Districts[1]

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Abstract

In many political jurisdictions, electoral districts are served by multiple representatives. In these multi-member district (MMD) contexts, elections pit incumbent legislators not only against challengers from rival parties, but also other incumbents in the same district, including co-partisan incumbents. We develop a formal theory of legislative representation in MMD systems, in which legislators trade off the pursuit of collective goals versus cultivating personal reputations. We unearth contexts in which MMD electoral systems can more effectively balance the interests of voters and parties as competing principals, relative to single-member districts (SMD). Our framework allows us to unify and re-examine a raft of existing theoretical and empirical claims about the consequences of proportional representation, and further derive new and testable empirical hypothesis about legislative cohesion across different MMD electoral rules.

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Understanding how electoral institutions affect policy and political outcomes is a defining goal of comparative politics. A vast scholarly literature studies how differences in electoral rules can account for cross-country variation in government formation (Austen-Smith and Banks (1988), Baron and Diermeier (2001)), income redistribution (Austen-Smith (2000), Persson and Tabellini (2005), Lizzeri and Persico (2001)), voting behavior (Cox and Shugart (1996), Cho (2014)), party systems (Duverger (1959), Morelli (2004)), and turnout (Blais and Carty (1990)). Experimentation with alternative electoral rules is ongoing across the world's old and young democracies.[1] And, new data is expanding scholars' abilities to link the properties of electoral system to representatives' behavior (Folke, Persson and Rickne (2016) and Raffler (2016)).

Electoral rules are typically classified as *majoritarian* or *proportional*, the key difference being the accuracy with which vote shares are translated into seat shares. This fundamental distinction guides most existing studies. Nonetheless, electoral systems also vary on other important dimensions. Most notably, they vary in whether voters choose a *single representative* or *multiple representatives*, usually drawn from competing *party lists*. In this paper, we propose a new theoretical framework to investigate how ballot structures shape representation from positive (e.g., legislative cohesion) and normative (e.g., accountability and selection) perspectives.

While multi-member districts are common than single member districts in legislatures around the world, surprisingly little is known about how they influence the accountability mechanism between voters and elected representatives. A recent report by the American Political Science Association's *Task Force on Electoral Rules and Democratic Governance* argued that existing work "fail[s] to identify the causal mechanism responsible for generating incentives for voters, parties, and individual legislators" (Shugart (2013)). Similarly, André, Depauw and Shugart (2013) conclude that the literature has been "largely unsuccessful in identifying the underlying causal mechanism tying the formal properties of the electoral institutions to the behavior of legislators."

We illustrate both the mechanics and strategic issues in multi-member contexts with a simple example. A district represented by two legislators in the national parliament, in which each of two political parties—the Incumbent and the Opposition—offers two candidates on a single *ordered* ballot—the party's *ticket*. Suppose that voters cast their votes for one or the other ticket, resulting in a vote share $\pi_I > .5$ for the Incumbent ticket, and $\pi_O = 1 - \pi_I < .5$ for the Opposition ticket.

[1] A notable example is Iraq, which varied its multi-member electoral system between open- and closed-party lists in every one of its three parliamentary elections between 2005 and 2010.

By virtue of winning a majority of votes, the Incumbent party wins the first seat. The electoral rule, however, must also specify an allocation mechanism for the second seat: the majority-winning party's vote share must exceed the other party's share by a sufficient margin: $\frac{\pi_I}{1+\rho} \geq \pi_O$. The parameter $\rho \geq 0$ represents the *electoral quota*. In real-world contexts, $\rho = 1$—i.e., a single party claims *both* seats only if its vote share exceeds two thirds)—is called the *D'Hondt* method, and it is used in legislatures in Argentina, Brazil, Colombia, Denmark, Iceland, Israel and Spain. The variant with $\rho = 2$ is called the *Sainte-Laguë* method, and is used in Norway, Iraq, Sweden and Germany. More broadly, a higher quota increases the minimum excess support that a party needs in order to claim each additional seat.

If the Incumbent party fails to attain this excess support, the single seat is awarded to the first-ranked candidate in the Incumbent ticket, even if the second-ranked candidate was more popular (and able to defeat her in a pair-wise majority vote). The presence of the list, therefore, has two key consequences: first, it transfers control over legislators' electoral incentives from voters to political party leaderships, who control the ballot order; second, it forces voters to use their ballot to express preferences not over individual politicians, but over *legislative teams.*

Crucially, a party's list ordering only affects the electoral fate of its candidates when the party's performance lies in an intermediate range: If the Incumbent ticket wins a vote share in excess of $\frac{1+\rho}{2+\rho}$ (that is, the Opposition ticket performs very poorly), *both* the first- and second-ranked Incumbent candidates are elected. Alternatively, if the Incumbent ticket wins a vote share below $\frac{\rho}{2+\rho}$, *neither* candidate is elected. An increase in the quota ρ expands the intermediate range of performance in which a party gains exactly one seat, and thus increases the linkage between an incumbent's rank on the ticket and her re-election prospects.

The performance of the ticket, however, also depends on the behavior of the politicians in office. Their incentives to act in ways that improve the ticket's overall success—and thus both politicians' prospect of reelection—or, instead, her own prospect of reelection, change with their position in the list. A first-ranked incumbent who is near-certain of reelection may feel little need to promote her voters' interest. Similarly, a lower-ranked candidate who anticipates a very low prospect of reelection may have little incentive to work for her constituents. While it seems natural to conjecture that multi-member districts weaken the accountability mechanism between representatives and constituents (e.g., Shugart (2005), Bawn and Thies (2003), Kunicova and Rose-Ackerman (2005)), our analysis shows that this conjecture is not always correct.

Our approach. We propose a formal-theoretic framework to study ballot structure shapes representatives' incentives balance the interests of their constituents and the goals of their party leaderships. To provide the starkest comparison between different accountability structures, we focus on closed lists, the strongest form of party control over candidates' electoral prospects.

Our theory builds upon two central ideas from the comparative politics literature. First, representatives are subject to two distinct sources of accountability: voters, and political party leaders. As elaborated in the seminal works by Hix (2002) and Carey (2007), the party leadership and district voters may act as *competing principals*: occasionally party leaders may want individual representatives to support policies that negatively affect their constituents. Second, multi-member districts create a tension between the value of *individual* and *collective* reputation (Carey and Shugart (1995), Shugart, 2013): actions that enhance an individual reputation might not always advance the ticket's collective reputation.

In line with our example above, our model features a continuum of local voters, two Incumbent representatives facing two Opposition candidates, and a national party leadership with a broad policy agenda. Voters rely on their representatives to act in their interest by choosing when to toe the party line, and when instead to oppose it.

Voters, however, face two fundamental deficiencies in their information: whether the party leadership's agenda benefits their local interests, and whether their representatives are intrinsically *aligned*, i.e., share constituents' primitive policy preferences, or *mis-aligned*, i.e., are willing to sacrifice constituency interests in order to foster their party's legislative accomplishment—for example, due to ideological considerations.

As a result, voters use a single source of data—legislators' voting records—to form an assessment about the consequences of the party's policy agenda *and* about each of their representatives' alignment. Crucially, voters' inferences about a representative is shaped not only by her own voting behavior, but also by that of her colleagues: a representative who supports a bill that many others are seen to oppose may be perceived differently than when she supports a bill that rallies support across her party. We study how ballot structure shapes the linkage between each representative's behavior, her individual reputation vis-a-vis voters and party leaders, the *collective reputation* of the party's ticket, and thus politicians' electoral survival.

Single-member districts establish a direct accountability relationship between voters and their elected representatives, uncontaminated by party leaderships. While allowing voters to separately hold each representative accountable for her behavior, they create an incentive for *obstructionism*: representatives' tendency to oppose the policy agenda even when it actually benefit the voters in order to foster their reputation. In this context–and contrary to prevailing scholarly wisdom[2]—we show that voters can indeed benefit from (i) ceding part of their electoral control to political party leaders with which they may be mis-aligned, and (ii) constraining their ability to cast a single ballot for a *bundle* of politicians.

The effect of multi-member districts on accountability depends on *how much control* voters relinquish. When the voter does not retain enough control, the electoral process features *rubber-stamping*: representatives' tendency to support the policy agenda even when it is detrimental to the constituency. We show how this tendency varies with primitives such as party polarization, the initial perceived alignment of incumbents and challengers, and the quota ρ. Our focus on the quota is especially novel: existing work typically develops predictions with respect to changes in district magnitude, i.e., the number of seats in the constituency (for example, Carey and Shugart (1995), Crisp, Jensen and Shomer (2007) and Norris (2006)). We show that rubber-stamping becomes more pervasive as the quota ρ increases. As our example clarifies, an increase in ρ expands the intermediate range of performance in which the party can retain only one seat. Hence, it strengthens the linkage between electoral survival, list ranking, and a reputation for alignment with the leadership.

In addition to trying to avoid punishment for defection (i.e., being second-ranked), multi-member districts produce an additional effect tempering representatives' tendency to obstructionism: Cultivating an *individual* reputation with voters may actually undermine the *collective* reputation of the party. While variants of the list proportional rule allow voters to express some degree of individual preference, their vote nonetheless always advances the cause of *all* politicians on the same party list.[3] By obstructing her party's agenda, a representative makes her co-partisans that support the policy look like a pusillanimous party puppet. Not only does this worsen her standing inside the party: it might actually worsen voters' evaluation of the entire ticket.

Our analysis highlights how different ballot structures determine whether an individual representative's re-election concerns are primary driven by *intra*- or *inter*-party competition (Katz (1985); Caillaud and Tirole (2002)). While single member districts encourages an aligned representative to speciously oppose projects that generates a positive surplus for the constituents, they mitigate the value to a mis-aligned representative from supporting

[2]For example, Shugart (2005) argues: "when party leaders control the prospects of elections of rank-and-file legislators, they can shield members and themselves from electoral accountability and provide favors to interest groups that lack a popular constituency."

[3]We will focus on closed-list electoral rules, in this paper. In practice, there is a plethora of flexible-list rules that allow voters to support individual politicians, and thus encourages them to build a personal vote, as conceptualized by Bruce, Ferejohn and Morris (1997). Nonetheless, votes cast for an individual politician do not guarantee her election, conditional on her party winning one or more seats. Thus, while closed lists highlight the collective externality in its most stark form, it is a common property of all list-based electoral rules.

projects that generate a negative surplus.

We develop a raft of observable implications, some of which are consistent with existing evidence, and some of which constitute opportunities for future empirical work. We predict a greater degree of legislative cohesion under closed-list multi-member districts than single-member settings, consistent with findings by Carey (2007), Depauw and Martin (2009), and we also predict that his cohesion will be larger in multi-member contexts with a larger electoral quota ρ. We further predict that greater polarization in the electorate will increase party discipline in multi-member districts, but not in single-member districts. We also show that political selection (i.e., the ability to screen out misaligned types) is always higher under single member districts *regardless of the quota*. And, we show that this result is driven *not* by party leaders' manipulation of the list: it arises because voters in multi-member districts are endogenously more responsive to partisanship and other factors that are unrelated to beliefs about alignment. We also show that multi-member districts can generate superior accountability between politicians and voters, but only if the electoral rule is not *too* proportional, i.e., if ρ is positive but not too large. We connect this result to a distinct but related claim by Carey and Hix (2011) that proportional representation combined with low district magnitude is the 'electoral sweet spot'.

The remainder of this paper is organized as follows. We present our baseline model, and discuss its relation to existing work on proportional rules. We then proceed with the analysis of single-member districts, before moving to the analysis of multi-member districts. We then use our results to generate positive predictions about the effect of primitives on legislative unity and legislative behavior, more generally. We compare the value of single- and multi-member districts for aligning the incentives of legislators with their constituency voters, and the relative prospect that voters can achieve the best possible means for selecting aligned politicians. Proofs and additional results are in an Appendix.

1. Model

Agents. We consider a two-date interaction between a continuum of voters (to whom we reserve the pronoun "she"), two incumbent co-partisan legislators (A and B), and their party leadership (L). The legislators represent the same geographic constituency. The party leadership represents an individual or collective agent with legislative agenda-setting authority. For example, it could be the leader of the majority party, the head of the executive in presidential settings, or a collective agent such as the cabinet in parliamentary settings. In addition to these players, there are two opposition politicians that may replace the incumbent legislators at the end of the first period. At each date, there is a *legislative interaction*, and between the first and second date there is an *election*.

Legislative Interaction. First, the leadership proposes a policy agenda for approval by the legislative body. For example, the policy could be a major public infrastructure program, a broad economic reform package, or an international agreement. While the leadership positively values the policy agenda, it is uncertain whether the constituency will also benefit from it. Specifically, the constituency payoff from the project is a random variable θ, uniformly distributed on $[-\kappa, \kappa]$. This value is privately learned by each representative, while the party leadership and the voters do not observe it.[4] If the policy is not implemented, all agents derive a status-quo policy payoff that we normalize to zero.

Second, each representative simultaneously supports or opposes the policy agenda by choosing either *aye* ($t = \mathsf{y}$) or *nay* ($t = n$). A *vote tally* $\mathsf{t} = (t_i, t_j) \in \{\mathsf{y}, \mathsf{n}\}^2$determines the probability that the agenda passes, according to $q(\mathsf{t})$, which satisfies:

$$q(\mathsf{y}, \mathsf{y}) > q(\mathsf{n}, \mathsf{y}) = q(\mathsf{y}, \mathsf{n}) > q(\mathsf{n}, \mathsf{n}). \tag{1}$$

Expression (1) states simply that the prospect that a bill passes is increasing in the number of votes in favor.

Election. After the leadership and voters observe the voting record of both incumbents (t), an election is held according to a process that we outline in further detail, below. The election determines whether each incumbent $i \in \{A, B\}$ keeps his seat ($e(i) = 1$), or instead loses it ($e(i) = 0$). We let $e = \{e(A), e(B)\}$

Under each electoral context, voters either vote for the incumbent party's ticket, or the opposition ticket. Voters compare the value from voting in favor of the incumbent ticket, V_I, and voting for the opposition ticket V_O. We defer the computation of these values since they are partly determined by the electoral context. Each voter j votes for the incumbent ticket if and only if:

$$V_I \geq V_O + \xi + \sigma_j, \tag{2}$$

where $\xi \sim U[-\frac{1}{2\psi}, \frac{1}{2\psi}]$, is an aggregate preference shock in favor of the Opposition party, and $\sigma_j \sim U[-\frac{1}{2\phi}, \frac{1}{2\phi}]$ is an individual-level preference shock in favor of the Opposition party. We interpret σ_j as an individual's partisanship. For example, voting decisions of agents who have very large σ_j or very small σ_j will be relatively insensitive to performance perceptions and will instead be driven by their attachment to the Opposition or Incumbent party. On the other hand, ξ reflects ephemeral factors that may influence all voters on polling day, such

[4]The assumption that the leadership does not observe θ is unimportant: all our results hold when the leadership learns θ. The crucial feature is that the opacity of parliamentary negotiations and the inherent uncertainty associated with policy outcomes generates substantial uncertainty on the party of voters about the local consequences of legislation.

as last-minute revelations of scandal or impropriety. It follows that the vote share accruing to the incumbent ticket is:

$$\Pi = \frac{1}{2} + \phi[V_I - V_O - \xi]. \tag{3}$$

Payoffs. A constituency voter's value from the policy agenda in each period is $q\theta$. Both incumbents and the leadership care about winning district-level elections and policy outcomes in both periods. The leadership's per period payoff is:

$$u_L(q, e) = qG + e(A) + e(B),$$

where q denotes the probability that the date-2 agenda is implemented, $G > 0$ denotes the elite's relative value of the policy, and $e(i)$ is incremental value of winning seats that does not directly relate to policy payoffs.[5]

Each politician values her own re-election, which yields a private office rent R. Politicians also care about policy outcomes. Each incumbent and opposition legislator may be *aligned* with her voters, or instead *mis-aligned* with her voters. We denote by μ the common prior that each incumbent is aligned and by μ_O the common prior that each opposition candidates is aligned. A politician's alignment determines her payoffs at each date associated with the outcome of the leadership's policy agenda:

$$u_i(q, e, \tau) = Re(i) + q(\theta + \tau).$$

For an aligned politician, $\tau = 0$: an aligned politician shares her constituency's policy preferences. By contrast, a mis-aligned politician derives an additional value $\tau \in \{-b, b\}$ in the event that a policy is passed. For a mis-aligned incumbent, $\tau = b > 0$, reflecting that the incumbent intrinsically values passing the leadership's agenda regardless of its consequences for her constituents. By contrast, a mis-aligned opposition politician derives the value $\tau = -b$ if the policy passes, reflecting a primitive hostility to the governing party's legislative agenda.

In addition to her direct payoffs from the implementation of the party's primary legislative agenda, each voter derives a per-capita incremental payoff $S > 0$ from having an aligned representative rather than a mis-aligned representative. S captures the myriad ways in which aligned representative can champion constituency interests beyond voting on the policy agenda. Examples include inserting favorable amendments into secondary bills, using gate-keeping powers in committee to promote certain bills and forestall progress on others, and delivering government contracts and grants to local firms and interests.

[5] We assume that the elite's value from the policy, G, is unrelated to the district-specific valuation θ. This eases presentation, but is not needed for our results.

Equilibrium. We study symmetric sequential equilibria. Note that the symmetry is with respect to the labeling of the incumbent politicians, *not* their alignment. Symmetry ensures that all sources of equilibrium asymmetry that arise in the behavior of different politicians is solely a consequence of the different incentives of aligned and mis-aligned politicians in different electoral contexts.

Discussion. We pause, briefly, to contrast our approach to the most closely related work. We also highlight aspects of real-world settings that we do not focus on, for parsimony, but which are explored in related work.

Candidate Selection and Recruitment. Our model endows party leaders with the ability to assign a list priority, but abstracts from party leaders' ability to recruit candidates (Carroll and Nalepa (2016)) or, more generally, the notion that the distribution of candidates' characteristics—and thus discipline—is endogenous to the legislative environment (Castanheira and Crutzen (2010)). Carroll and Nalepa (2016), in particular, argue that parties will recruit more ideologically homogeneous candidates to offset the higher incentive for personal vote-seeking (and thus, reduced discipline) in the absence of effective sanctions.

Other Sources of Party Discipline. In addition to the list ordering, party leaders have other means to incentivize the behavior of individual representatives, such as promotions, committee assignments, pork (Ames (1995), Crisp et al. (2004)) or other forms of beneficial transfers (Iaryczower et al. (2008)). In our setting, these tools would be of limited effectiveness to party leaders, especially under reasonable assumptions about resource constraints. Moreover, our notion of a congruent type captures, among other things, the susceptibility to this wide array of alternative disciplining tools.

Other Political Outcomes. We focus on legislative unity, as captured by legislators' roll-call votes. In practice, a legislator's degree of alignment with her constituency affects a variety of other outcomes such as her constituency service (e.g., Bowler and Farrell (1993)) and propensity to engage in corruption while in office (e.g., Chang (2005)). One may interpret the quantity S in our framework—a voter's value from an aligned representative that is unrelated to her roll-call vote—as reflecting these considerations.

2. Policy Outcomes at Date 2

We begin by deriving the voting behavior of representatives at the second (i.e., terminal) date. Recall that an aligned representative values the party agenda only inasmuch as it generates a positive surplus for his constituents, i.e., if $\theta > 0$. By contrast, a mis-aligned representative derives a value $\theta + \tau$ from the party agenda, where $\tau \in \{-b, b\}$: Specifically, a mis-aligned Incumbent derives a value $b > 0$ whenever the agenda is passed, and a mis-aligned Opposition legislator derives a value $-b < 0$ whenever the agenda is passed.[6] We therefore obtain:

Lemma 1. In the second period:
(i) an *aligned* representative votes *aye* on the party agenda if and only if $\theta > 0$;
(ii) a *mis-aligned* representative from a governing party votes in favor if and only if $\theta > -b$;
(ii) a *mis-aligned* representative from an opposition party votes in favor if and only if $\theta > b$.

This result highlights a tension between the value to voters and party elites from retaining aligned versus mis-aligned incumbent representatives. While voters always prefer an aligned representative, the incumbent leadership's second period policy agenda is more likely to succeed when legislators are relatively mis-aligned with their constituents, since they will vote in favor of policies $\theta \in [-b, 0]$ that aligned representatives would reject. We next study how these tensions are shaped by electoral institutions.

3. Single-Member Districts: Personal Reputation

Under single-member districts (SMD), the constituency is divided into two distinct electoral districts, each represented by a single legislator. An election is held in each district, in which the incumbent representative is pitted against an opposition challenger.

We show that there is a unique symmetric equilibrium, which is fully characterized by a pair of constituency threshold values for the project $(\underline{\theta}^S, \overline{\theta}^S)$: an *aligned* representative votes in favor of the project if and only if $\theta \geq \overline{\theta}^S$, and a *mis-aligned* representative votes in favor of the project if and only if $\theta \geq \underline{\theta}^S$.

Equilibrium thresholds. To derive properties of this threshold, let $p_i^S(\mathsf{t})$ denote the equilibrium probability that an incumbent i is retained in her district when the vote tally is $\mathsf{t} = (t_i, t_j) \in \{\mathsf{y}, \mathsf{n}\}^2$. Consider the problem faced by an *aligned* representative, who learns the constituency value of the policy θ and does not know the alignment of his co-partisan. In equilibrium, his expected relative value of voting in favor of the policy (y) at the threshold value $\overline{\theta}^S$ is:

$$R\left[p_i^S(\mathsf{y}, \mathsf{y}) - p_i^S(\mathsf{n}, \mathsf{y})\right] + \left[q(\mathsf{y}, \mathsf{y}) - q(\mathsf{n}, \mathsf{y})\right] \overline{\theta}^S. \tag{4}$$

Crucially, this relative value depends on what voters will infer about each representative's alignment from the observed vote tally. When both representatives support the leadership's agenda (y, y), voters understand that this outcome can arise from two different situations:

[6] The symmetry of the mis-aligned representatives' biases across parties does not play an important qualitative role in our analysis.

First, the project could be very valuable to the district, i.e., $\theta \geq \overline{\theta}^S$: in this case, both representatives vote in favor regardless of alignment. *Second*, the project could be sufficiently bad for the district that both representatives vote in favor only if they are both mis-aligned, i.e., $\theta \in [\underline{\theta}^S, \overline{\theta}^S]$. In the first event, two votes in favor of the party's agenda is no news about the representative's alignment; in the second event, two votes in favor are bad news about *both* incumbents' alignment. Letting $\hat{\mu}_i(\mathsf{t})$ denote the posterior belief about representative i's alignment after a vote tally $\mathsf{t} = (t_i, t_j)$, we have:

$$\hat{\mu}_i(\mathsf{y}, \mathsf{y}) = \frac{\mu}{1 + \frac{F(\overline{\theta}^S) - F(\underline{\theta}^S)}{1 - F(\overline{\theta}^S)}} < \mu, \tag{5}$$

where $F(\cdot)$ is the cumulative density of θ.

If the representative votes against the policy, by contrast, voters' inference about alignment reflects the following reasoning: if the project were sufficiently valuable to the district, i.e., $\theta \geq \overline{\theta}^S$, both representatives would vote *aye* regardless of their alignment; likewise, if the project were sufficiently costly for the district, i.e., $\theta \leq \underline{\theta}^S$, both representatives would vote *nay* regardless of alignment. A split vote implies that the project is not valuable enough to receive the support of an aligned representative ($\theta \leq \overline{\theta}^S$), but not so harmful to be opposed by a mis-aligned representative ($\theta \geq \underline{\theta}^S$). In that case, the representative that supported the project is surely mis-aligned, and the representative that opposed the project is surely aligned: $\hat{\mu}_i(\mathsf{n}, \mathsf{y}) = 1$. By the same reasoning, $\hat{\mu}_i(\mathsf{y}, \mathsf{n}) = 0$: if the voters' representative supports the project while the other votes against, voters interpret this record as evidence that their own representative is *mis-aligned*.

More generally, observing both representatives vote in favor of the leadership's policy is *always* bad news about both representatives' alignment—it raises voters' suspicion that their representatives are working with the leadership at the expense of their districts. By contrast, observing their own representative vote *nay* when the other representative votes *aye* fully assures voters that their own representative is aligned. In the single-member setting, incumbents do not compete directly with one another: each represents a different district and competes against a distinct challenger. Nonetheless, each representative's voting record may positively or negatively affect the other's prospect of re-election. The threshold $\overline{\theta}^S$ is obtained by setting the aligned representative's relative value of voting for the policy, given by expression (4), to zero.

We derive the cut-off value of the project $\underline{\theta}^S$ by a similar process. At the threshold $\underline{\theta}^S$, a mis-aligned representative reasons that his colleague will vote in favor if and only if he is mis-aligned, with probability $1-\mu$. Thus, his own relative value of voting in favor is:

$$R\left[\begin{array}{c}(1-\mu)\left(p_i^S(\mathsf{y},\mathsf{y})-p_i^S(\mathsf{n},\mathsf{y})\right)+\\+\mu\left(p_i^S(\mathsf{y},\mathsf{n})-p_i^S(\mathsf{n},\mathsf{n})\right)\end{array}\right]+\left[\begin{array}{c}(1-\mu)\left(q(\mathsf{y},\mathsf{y})-q(\mathsf{n},\mathsf{y})\right)+\\+\mu\left(q(\mathsf{y},\mathsf{n})-q(\mathsf{n},\mathsf{n})\right)\end{array}\right](\underline{\theta}^S+b), \qquad (6)$$

where we recall that $b>0$ captures the extent to which a mis-aligned representative favors his party's agenda above and beyond its intrinsic value to his district.

Probability of re-election. In single-member districts, each incumbent's probability of re-election is the probability that his vote share exceeds the challenger's, i.e., that her vote share exceeds one half. Using expression (3), we can write this prospect of winning:

$$p_i^S(\mathsf{t})=\frac{1}{2}+\psi[V_I^S(\mathsf{t})-V_O^S(\mathsf{t})]. \qquad (7)$$

where $V_I^S(\mathsf{t})$ is a voter's value from the incumbent ticket—in the single-member setting, the anticipated value of the having the incumbent in the second period–given a vote tally t. Notice that *both* the value of voting for the incumbent *and* the value of voting for an opposition party challenger depend on the vote tally. The reason is that the second period legislative voting outcome—which generates payoff consequences for the district—depends on the voting behavior of representatives across *both* districts.

For example, suppose that the voters observe a split vote in which their own representative A voted *aye*, but the other district's representative B voted *nay*. We earlier showed that, in this case, all voters infer that A is misaligned, while B is aligned. In the event that the other district retains their representative B, the value of retaining A is:

$$\mathbb{E}[\theta\geq 0]q(\mathsf{y},\mathsf{y})+\mathbb{E}[-b<\theta<0]q(\mathsf{y},\mathsf{n})+\mathbb{E}[\theta\leq -b]q(\mathsf{n},\mathsf{n}). \qquad (8)$$

If the constituency value of the second period agenda is $\theta>0$, both legislators will vote *aye* regardless of alignment. Likewise, if the constituency value of the second period agenda is $\theta<-b$, both legislators will vote *nay* regardless of alignment. If, instead, the constituency value of the second period agenda is $\theta\in(-b,0)$, A will vote *aye*—despite the negative payoff consequences for the district—while B will vote *nay*. Thus, if both representatives are retained, voters anticipate a split second-period vote tally whenever $\theta\in(-b,0)$.

Suppose, instead, that while the other district retains their representative B, voters in the district represented by the incumbent legislator A is replaced by the opposition party challenger, who is expected to be aligned with her district with probability μ_O. In that case, the district's value is:

$$\mu_O S + \mathbb{E}[\theta \geq b]q(\mathsf{y},\mathsf{y}) + \mathbb{E}[0 < \theta < b]\begin{bmatrix} \mu_O q(\mathsf{y},\mathsf{y}) \\ +(1-\mu_O)q(\mathsf{n},\mathsf{y}) \end{bmatrix} + \mathbb{E}[\theta < 0]q(\mathsf{n},\mathsf{n}). \tag{9}$$

If the constituency value of the second period agenda is $\theta > b$, both legislators will vote *aye* regardless of alignment. If the second period constituency value of the project is $\theta < 0$, both representatives will vote *nay*: The incumbent B votes against because he is aligned; the challenger that replaced incumbent A votes against regardless of her alignment, since the agenda hurts the district and voting against it also reduces the incumbent party's legislative success. If the constituency value is $\theta \in (0, b)$, legislator B will support the agenda (she would do so regardless of her alignment, since the project brings a positive surplus to the district). Support from the newly elected opposition legislator, however, is uncertain: If he, too, is aligned, he will also vote *aye*; if he is mis-aligned, instead, her desire to thwart the incumbent party's legislative agenda will induce her to vote *nay*, since $\theta < b$.

Analytic Approximation. The previous section highlights the construction of voters' values from retaining or replacing their politicians. Inserting these values in expression (7) yields the equilibrium probability with which each incumbent is retained; then, inserting these into expressions (4) and (6), and equating these expressions to zero yields the equilibrium thresholds $\overline{\theta}^S$ and $\underline{\theta}^S$. This exercise is complicated by the fact that there is no explicit analytic expression for these thresholds: they enter (1) directly into each legislator's first period payoff, (2) indirectly through the voter's inference (for example, (5)), and (3) directly into the voters' values from retention versus replacement. While we obtain a unique equilibrium, so long as κ is not too small, the solution of this highly nonlinear system is not tractable for analysis.

In order to facilitate comparisons across systems, we employ an analytic approximation in which we take the uncertainty κ associated with the consequences of the leadership's agenda for districts, to be large. In the Appendix, we show that (1) when κ is not too small, there is a unique pair of thresholds that satisfy the equilibrium conditions, and (2) as κ becomes large, these equilibrium thresholds converge to quantities that can be characterized analytically. We subsequently use the short-hands $\chi_2 = q(\mathsf{y},\mathsf{y}) - q(\mathsf{n},\mathsf{y})$ and $\chi_1 = q(\mathsf{y},\mathsf{n}) - q(\mathsf{n},\mathsf{n})$. So, χ_2 is the change in the prospect of the bill passing when the number of votes in favor reverts from two to one, and χ_1 is the change in the prospect that the bill passes when the number of votes in favor reverts from one to zero.

Proposition 1. Under single-member districts, for κ not too small, there exists a unique symmetric equilibrium. In this equilibrium, an aligned representative votes *aye* if and only if $\theta \geq \overline{\theta}^S(\kappa)$, and a misaligned representative votes *aye* if and only if $\theta \geq \underline{\theta}^S(\kappa)$, where:

$$\lim_{\kappa \to \infty} \overline{\theta}^S(\kappa) \equiv \overline{\theta}^S = \frac{\psi RS}{\chi_2}(1-\mu), \tag{10}$$

$$\lim_{\kappa \to \infty} \underline{\theta}^S(\kappa) \equiv \underline{\theta}^S = \frac{\psi RS}{(1-\mu)\chi_2 + \mu\chi_1}(1-2\mu(1-\mu)) - b. \tag{11}$$

Cultivating a Personal Reputation. We now explore in more detail how the incentive to cultivate a personal reputation in single-member districts generates the thresholds we derived in Proposition 1.

In the single-member context, the incumbent ticket consists of a single representative, who is evaluated relative to a challenger. In the Appendix, we show that as uncertainty about the value of the party agenda κ grows large, voters' value from retaining the incumbent representative $i \in \{A, B\}$ converges to their posterior beliefs about his alignment, $\mu_i(\mathsf{t})$, where $\mathsf{t} \in \{\mathsf{y}, \mathsf{n}\}^2$ is the first-period vote tally. Notice that voters use the vote tally of *both* representatives in order to form an evaluation of their own representative's alignment. Suppose, for example, that representative A considers voting against an agenda that generates a surplus $\overline{\theta}^S$. In equilibrium, she conjectures that her co-partisan—regardless of alignment—will vote in favor. If both vote in favor, i.e., the vote tally is $\mathsf{t} = (\mathsf{y}, \mathsf{y})$, (5) implies that voters' believe that the prospect of their representative being aligned is:

$$\mu_A(\mathsf{y}, \mathsf{y}) = \frac{\mu}{1 + \frac{F(\overline{\theta}^S) - F(\underline{\theta}^S)}{1 - F(\overline{\theta}^S)}}. \tag{12}$$

If, instead, representative A votes against the policy, the vote tally is $\mathsf{t} = (\mathsf{n}, \mathsf{y})$; voters infer that their representative is aligned (and that representative B is mis-aligned), i.e.:

$$\mu_A(\mathsf{n}_A, \mathsf{y}_B) = 1. \tag{13}$$

Thus, the change in the incumbent A's prospect of re-election from defecting to a vote against the agenda when she believes that her legislative co-partisan will support, is the difference of expressions (13) and (12); as κ grows large, this equilibrium difference converges to $1 - \mu$, which is the term that appears in the threshold for the aligned type in Proposition 1.

The threshold for the mis-aligned representative follows a similar derivation. However, at the threshold $\underline{\theta}^S$, the mis-aligned representative is uncertain of her co-partisan's vote: in equilibrium, she anticipates that her colleague will support the leadership's agenda if and only if she is also mis-aligned, with probability $1 - \mu$. Thus, her expected difference in her posterior evaluation with her own voters when she votes against the agenda is:

$$\mu(\mu_B(\mathsf{n}_B, \mathsf{n}_A) - \mu_B(\mathsf{y}_B, \mathsf{n}_A)) + (1 - \mu)(\mu_B(\mathsf{n}_B, \mathsf{y}_A) - \mu_B(\mathsf{y}_B, \mathsf{y}_A)),$$

and as κ grows large this equilibrium difference converges to:

$$\mu(\mu - 0) + (1 - \mu)(1 - \mu) = 1 - 2\mu(1 - \mu).$$

as reflected in the threshold for the mis-aligned type in the Proposition.

We advance some observations about these thresholds. First, $\overline{\theta}^S > 0$: an aligned representative votes only for an agenda that generates a positive surplus for his district. However, there is a measure of positive surplus agendas $\theta \in [0, \overline{\theta}^S]$ that aligned representatives oppose. This arises from incentives to cultivate a personal reputation, i.e., to pander: voting *nay* rather than *aye* generates a more favorable personal reputation with district voters, who reward the representative with a higher prospect of re-election.

By the same reasoning, $\underline{\theta}^S > -b$: mis-aligned incumbents also oppose agendas that they would nonetheless prefer to pass. Moreover, if office-holding motives are sufficiently potent, $\underline{\theta}^S > 0$: a mis-aligned representative votes only for an agenda that generates a positive surplus for his district. In this case, a mis-aligned representative more faithfully represents his constituency interest than an aligned representative, since $0 < \underline{\theta}^S < \overline{\theta}^S$.

More generally, incumbents in single-member districts have an incentive solely to cultivate a *personal reputation* for alignment with their constituency voters. While neither representative stands in direct competition with the other, each recognizes that voters in *both* districts form inferences about their representative's alignment based on the *entire* vote tally. If incumbent A expects incumbent B to support the agenda, voting against it sends his voters a powerful signal that he is willing to cross party lines in order to oppose policies that are harmful for the district; if, instead, A expects B to oppose the agenda, voting in favor sends his voters a powerful signal that he is a party stooge who puts the party's legislative success ahead of his constituents. Both forces encourage a representative to obstruct and oppose.

To summarize: with single-member districts, both *aligned* and *mis-aligned* representatives reject policies that they would have preferred to implement absent re-election incentives. An aligned representative always rejects policies that voters would have preferred her to support, since $\overline{\theta}^S > 0$. If office-holding motives are sufficiently potent, a mis-aligned representative also displays similar obstructionist behavior.

4. Multi-Member Districts: Personal *and* Collective Reputation

We now turn to the analysis of legislative behavior and elections in multi-member districts (MMD). While MMD electoral systems vary across many important dimensions, we focus on *closed-list* electoral rules. Our motivation is partly empirical: the majority of legislatures elected with proportional representation elect their members using closes lists, including Argentina, Israel, Italy, Iraq, Spain, South Africa and Turkey. An additional motivation for focusing on closed list is the substantive focus of our paper on competing principals: since closed list electoral rules most starkly vest control of incumbents' electoral fortune in the hands of political party leaderships, it would seem to be the most natural starting point for comparing SMD and MMD systems along this dimension.

Under (closed-list) MMD, the constituency forms a single electoral district, which is represented by two legislators. After the first period vote, but prior to the election, the party leadership L constructs the party *list* assignment, either $\{AB\}$ or $\{BA\}$. The list denotes the order in which seats are filled after the election: if the list is $\{AB\}$ and the party wins only a single seat, the seat is awarded to representative A; if the list is $\{BA\}$ and the party wins only a single seat, the seat is awarded to representative B. Of course, if the party wins both seats, both representatives are re-elected regardless of their individual positions in the list.

Voters observe the list and vote either for the incumbent ticket—i.e., the closed list put up by the incumbent party—or the opposition ticket. Using expression (3), we can write the vote share won by the incumbent party as:

$$\Pi(\mathsf{t}, \{ij\}) = \frac{1}{2} + \phi[V_I^M(\mathsf{t}, \{ij\}) - V_O^M(\mathsf{t}, \{ij\}) - \xi], . \tag{14}$$

Note that we index the values of voting for either ticket by the electoral system, reflecting the fact that the value of voting for the incumbent ticket under MMD is different than under SMD. Under SMD, the prospect that a representative is re-elected is simply the prospect that the incumbent ticket receives a vote share in excess of fifty percent. Under MMD, the condition for re-election depends on the representative's standing in the party list, since a party may win zero, one, or two seats. Consistent with real-world closed list settings, seats are allocated according to the following procedure.

1. The first seat is awarded to whichever party list receives the highest vote share. That is, the incumbent ticket is awarded the first seat if $\Pi \geq \frac{1}{2}$; the opposition ticket is awarded the first seat if $\Pi < \frac{1}{2}$.

2. If the incumbent party wins the first seat, it further wins the second seat only if

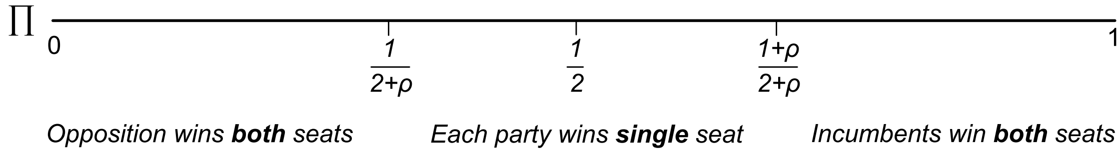

Figure 1: How the vote share of the incumbent ticket, Π, determines seat allocations in multi-member districts.

$\frac{\Pi}{1+\rho} > 1 - \Pi$; likewise, if the opposition party wins the first seat, it further wins the second seat only if $\frac{1-\Pi}{1+\rho} > \Pi$.

Figure 1 illustrates. The parameter $\rho \geq 0$ is the electoral *quota*, and reflects the proportionality of the system. In real-world contexts, $\rho = 1$ is called the *D'Hondt* method, and it is used in legislatures in Argentina, Brazil, Colombia, Denmark, Iceland, Israel and Spain. The variant with $\rho = 2$ is called the *Sainte-Laguë* method, and it is used in Norway, Iraq, Sweden and Germany. A higher value of ρ increases the amount of excess support that one party has to gather over the other in order to claim the second seat. For example, when $\rho = 1$, the party that wins a majority of votes wins *both* seats only if its vote share further exceeds two-thirds; when $\rho = 2$, the requirement is raised to a vote share in excess of three quarters. It is important to notice that MMD are not inherently more proportional than SMD: the limiting case of $\rho = 0$ corresponds to Party Bloc Voting, in which the plurality winning list obtains *all seats*.[7]

We show that there is a unique symmetric equilibrium. As in single-member districts (SMD), the equilibrium is characterized by threshold values $\underline{\theta}^M$ and $\overline{\theta}^M$ such than an *aligned* representative votes in favor of the policy agenda if and only if $\theta \geq \overline{\theta}^M$, and a *mis-aligned* representative votes in favor of the policy agenda if and only if $\theta \geq \underline{\theta}^M$.

We start by deriving a representative's prospect of re-election, which depends both on the the electorate's voting behavior and the party leadership's choice of a list order.

Probability of re-election Putting aside, for now, the computation of equilibrium values ${V_I}^M(\mathtt{t})$ and ${V_O}^M(\mathtt{t})$ (which we develop as part of our argument in the proof of Lemma 2), we may write the probability that incumbent $i \in \{A, B\}$ is retained when assigned the first position on the party list and the probability of retention when assigned the second position

[7] Currently the system is only used in a handful of countries, among which Singapore.

on the list:

$$p_i{}^M(\mathsf{t},\{ij\}) = \frac{1}{2} + \frac{\psi}{2\phi}\frac{\rho}{\rho+2} + \psi(V_I{}^M(\mathsf{t},\{ij\}) - V_O{}^M(\mathsf{t},\{ij\})) \tag{15}$$

$$p_i{}^M(\mathsf{t},\{ji\}) = \frac{1}{2} - \frac{\psi}{2\phi}\frac{\rho}{\rho+2} + \psi(V_I{}^M(\mathsf{t},\{ji\}) - V_O{}^M(\mathsf{t},\{ji\})). \tag{16}$$

Notice, first, that the prospect of retention for a first-ranked incumbent is *increasing* in ρ, the progressivity of the electoral rule, but that the prospect of retention for a second-ranked incumbent is *decreasing* in ρ. The reason is that as ρ rises, both the incumbent and the opposition ticket are increasingly likely to gain at least one seat. In turn, receiving the highest position on the list becomes more likely to be *both* necessary *and* sufficient for re-election.

In the proof of Lemma 2, we show that the probability of being pivotal for the awarding of each of the two possible seats is the same.[8] As a result, $V_I{}^M(\mathsf{t},\{ij\})$ and $V_O{}^M(\mathsf{t},\{ij\})$ weigh these two events with equal probability, and are derived in same way as the values in expressions (8) and (9).

Leadership choice. We begin by identifying how the leadership's list assignment responds to the vote tally, $\mathsf{t} \in \{\mathsf{y},\mathsf{n}\}^2$. If both representatives vote *aye*, i.e., $\mathsf{t} = (\mathsf{y},\mathsf{y})$, or both representatives vote *nay*, i.e., $\mathsf{t} = (\mathsf{n},\mathsf{n})$, we assume that the leadership's priority is determined by a fair coin toss.

Lemma 2. If representatives' votes differ from each other, the leadership strictly prefers to award electoral priority to the representative that votes *aye*: the leadership chooses $\{AB\}$ if $(t_A, t_B) = (\mathsf{y},\mathsf{n})$ and $\{BA\}$ if $(t_A, t_B) = (\mathsf{n},\mathsf{y})$.

Equilibrium thresholds. We now derive the equilibrium thresholds. Consider the problem faced by an aligned representative $i \in \{A, B\}$, who learns that constituency value of the policy θ and does not know the alignment of his co-partisan. In equilibrium, his expected relative value of voting in favor of the policy (y) at the threshold $\overline{\theta}^M$ is:

$$R\left[\frac{1}{2}(p_i{}^M(\mathsf{y},\mathsf{y},\{ij\}) + p_i{}^M(\mathsf{y},\mathsf{y},\{ji\})) - p_i{}^M(\mathsf{n},\mathsf{y},\{ji\})\right] + \chi_2\overline{\theta}^M. \tag{17}$$

If both representatives vote *aye*, so that the vote tally is (y,y), the incumbent representative i anticipates that the leadership is equally likely to assign her the priority, so that

[8] In a Supplemental Appendix, we develop alternative models of voter behavior, and show that our results qualitatively extend. We are grateful to Stephane Wolton for encouraging us to pursue these approaches, and for suggesting one.

the ballot order is $\{ij\}$, or instead assign the priority to her co-partisan, so that the ballot order is $\{ji\}$. Alternatively, the incumbent representative i could vote against the policy, anticipating that her fellow representative will vote in favor. The split record (n, y) ensures that the leadership assigns her the lowest rank in the list, i.e., chooses the ballot order $\{ji\}$.

Similarly, the expected relative value of voting in favor of the policy (y) for a mis-aligned incumbent at the threshold $\underline{\theta}^M$ is:

$$R\left[\begin{array}{c}(1-\mu)\left(\frac{1}{2}(p_i^M(\mathsf{y},\mathsf{y},\{ij\})+p_i^M(\mathsf{y},\mathsf{y},\{ji\}))-p_i^M(\mathsf{n},\mathsf{y},\{ji\})\right)\\ +\mu\left(p_i^M(\mathsf{y},\mathsf{n},\{ij\})-\frac{1}{2}(p_i^M(\mathsf{n},\mathsf{n},\{ij\})+p_i^M(\mathsf{n},\mathsf{n},\{ji\}))\right)\end{array}\right]+\left[\begin{array}{c}(1-\mu)\chi_2\\ +\mu\chi_1\end{array}\right](\underline{\theta}^M+b). \tag{18}$$

Employing the same analytic approximation that we used for the case of single-member districts, we obtain the following result.

Proposition 2. Under multi-member districts, for κ not too small, there exists a unique symmetric equilibrium. In this equilibrium, an aligned representative votes *aye* if and only if $\theta \geq \overline{\theta}^M(\kappa)$, and a misaligned representative votes *aye* if and only if $\theta \geq \underline{\theta}^M(\kappa)$, where:

$$\lim_{\kappa\to\infty}\overline{\theta}^M(\kappa)\equiv\overline{\theta}^M=\frac{\psi RS}{2\chi_2}\left[\frac{1}{2}-\mu-\frac{1}{S\phi}\frac{\rho}{\rho+2}\right], \tag{19}$$

$$\lim_{\kappa\to\infty}\underline{\theta}^M(\kappa)\equiv\underline{\theta}^M=\frac{\psi RS}{2((1-\mu)\chi_2+\mu\chi_1)}\left[2\left(\frac{1}{2}-\mu\right)^2-\frac{1}{S\phi}\frac{\rho}{\rho+2}\right]-b. \tag{20}$$

To understand the result, consider first the expression for $\overline{\theta}^M$, and in particular the term in brackets, which consists of two distinct incentives.

Personal Reputation: The prospect that the incumbent ticket wins *exactly one* seat is:

$$\frac{\psi}{\phi}\frac{\rho}{\rho+2},$$

which *increases* in ρ, since a more proportional electoral rule is more likely to assign both the Incumbent and Opposition ticket a single seat. This raises the value of cultivating favor with the party leadership: being assigned the electoral priority by the party leadership is more likely to be both necessary and sufficient for re-election. And, we earlier showed that the leadership always prefers to assign the electoral priority to the incumbent that is most likely to be mis-aligned with voters. This creates a powerful incentive for both an aligned and a mis-aligned representative to vote in favor of the first-period agenda.

As in the single-member context, incumbents in multi-member districts also value their personal reputation, but instead of seeking to cultivate a personal reputation for alignment with the interests of his *constituents*, each tries to cultivate a personal reputation for alignment with the interests of his *party leadership*!

Corollary 1. (i) *As electoral system proportionality ρ increases, both aligned and mis-aligned representatives are more likely to vote in support of their party agenda: $\overline{\theta}^M$ and $\underline{\theta}^M$ decrease in ρ.*

Collective Reputation: In *single-member districts*, the Incumbent ticket consists of a single representative. Relative to the Opposition candidate, a representative is assessed solely on the basis of her own perceived alignment. While she may vote together with her parliamentary colleagues, she competes on her own.

In *multi-member districts*, by contrast, the Incumbent ticket consists of more than one representative. Relative to the Opposition ticket, a voter who considers her value from supporting the Incumbent ticket must form an evaluation of *the whole team*, since she cannot direct her vote solely to either representative.

In the Appendix, we show that the value voters derive from the Incumbent ticket in multi-member systems is a convex combination of their posterior belief about each incumbent's alignment. With uniform preference shocks (ξ), each posterior is weighted equally; as uncertainty about the party's agenda grows large, voters' value converges to:

$$\frac{\mu_A(\mathsf{t}) + \mu_B(\mathsf{t})}{2}. \tag{21}$$

Unlike single-member districts, each representative's prospect of re-election depends *directly* on *both* incumbents' evaluation. Suppose, for example, that representative A considers voting against an agenda that generates a surplus $\overline{\theta}^M$. In equilibrium, she conjectures that her co-partisan—regardless of alignment—will vote in favor. If both vote in favor, i.e., the vote tally is $\mathsf{t} = (\mathsf{y}, \mathsf{y})$, (21) implies that the voter's value from the incumbent team is:

$$\frac{\mu_A(\mathsf{y},\mathsf{y}) + \mu_B(\mathsf{y},\mathsf{y})}{2} = \mu(\mathsf{y},\mathsf{y}) = \frac{\mu}{1 + \frac{F(\overline{\theta}) - F(\underline{\theta})}{1 - F(\overline{\theta})}}. \tag{22}$$

If, instead, representative A votes against the policy, the vote tally is $\mathsf{t} = (\mathsf{n}, \mathsf{y})$; voters infer that representative is aligned, and representative B is mis-aligned, and thus derive the following value from the incumbent team:

$$\frac{\mu_A(\mathsf{n},\mathsf{y}) + \mu_B(\mathsf{n},\mathsf{y})}{2} = \frac{1+0}{2} = \frac{1}{2}. \tag{23}$$

This expression reveals that despite cultivating a favorable personal reputation with voters, representative A partially bears the cost of her defection; her gain in *personal reputation* comes at the cost of the ticket's *collective reputation.* Since the voter votes on the entire ticket, rather than individual representatives, representative A is forced to bear some of the costs of her attempt to build personal reputation at the expense of her co-partisan. As κ becomes arbitrarily large, the difference between expressions (23) and (22) converges to $\frac{1}{2} - \mu$, which is the first part of the bracketed term in the threshold $\overline{\theta}^M$.

The net change in collective reputation resulting from a split voting record therefore depends on the initial reputation of the legislative team, μ. If legislators are initially held in high regard, i.e., $\mu > .5$, the net change is *negative*: the gain in personal reputation is dominated by the loss in collective reputation. If, however, legislators are initially viewed poorly, i.e., $\mu < .5$; the net gain is *positive*: the gain in personal reputation generates a positive externality for the remaining legislator.

5. Legislative Cohesion, Political Selection and Accountability Across Systems.

We now turn to a comparison of single- and multi-member districts on both positive and normative grounds, and illustrate how these comparisons vary with changes in primitives.

Legislative Cohesion. We begin by highlighting the key differences in legislators' voting behavior across these systems. In particular, we show that, *regardless* of other primitives such as polarization (ϕ), the relative importance of party identification in the electorate (ψ), and the extent to which mis-aligned representatives intrinsically value their party's agenda (b), both *aligned* and *mis-aligned* legislative representatives have a higher propensity to support the party's legislative agenda in multi-memberdistricts, relative to single-member districts.

Proposition 3. Both *aligned* and *mis-aligned* representatives are more prone to supporting the party's agenda in multi-member districts than in single-member districts: $\overline{\theta}^M < \overline{\theta}^S$ and $\underline{\theta}^M < \underline{\theta}^S$.

The Proposition confirms that—relative to a system in which political accountability is enforced solely by voters—a system that shares responsibility between parties and voters diminishes incumbents' value from cultivating a favorable personal reputation with voters, i.e., pandering. Note that the Proposition does, in and of itself, imply that one system is superior to the other: where single-member districts generate a problem of too few bills being passed, i.e., projects with positive surplus $\theta > 0$, the next corollary notes that multi-member districts may be similarly liable to result in to many bills being passed, i.e., projects with negative surplus $\theta < 0$.

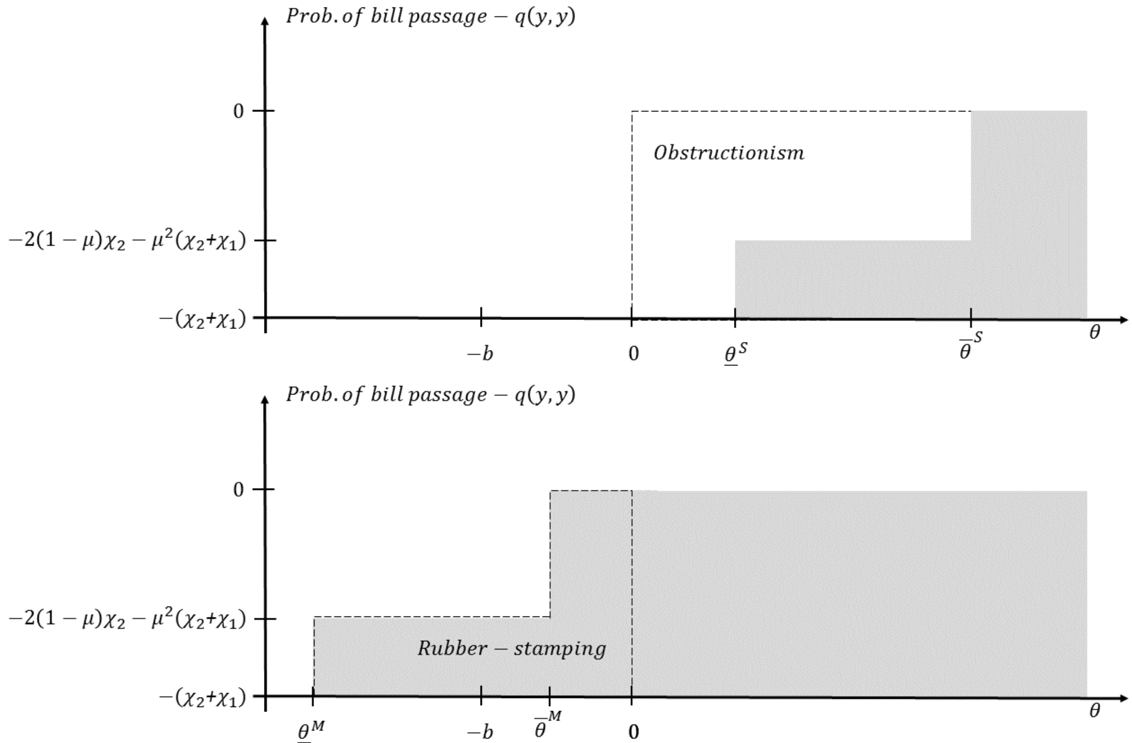

Figure 2: Ex-ante probability of bill approval (relative to $q(y, y)$), as a function of the local net benefit θ under Single Member Districts (SMD) and Multi-member districts (assuming condition (24) holds. The gray area indicates positive probability of passage. Under SMD, some bills yielding a positive payoff to the constituency are not passed. Under MMD, some bills yielding a negative payoff to the constituency are passed (with probability one, when the consequences are negative but limited).

Corollary 2. An *aligned* representative in a single-member district always vetoes projects that generate a negative surplus for his district in single-member districts, i.e., $\overline{\theta}^S > 0$. By contrast, an *aligned* representative in a multi-member district approves some negative surplus projects, i.e., $\overline{\theta}^M < 0$, whenever $\mu > .5$, or if $\mu \leq .5$ and the electoral system is sufficiently progressive, i.e.:

$$\rho \geq \frac{2S\phi(.5 - \mu)}{1 - S\phi(.5 - \mu)} = \hat{\rho}(\phi, \mu). \tag{24}$$

The corollary illustrates how the proportionality of the electoral rule, ρ, is fundamental to the comparison of electoral environments. Recall that higher values of ρ imply that the vote wedge between the first- and second-ranked party must be ever-larger in order for both seats to be awarded to the first-ranked party.

As ρ increases, a first-ranked position on the party list is increasingly likely to be both necessary and sufficient for an incumbent's re-election. The reason is that higher values of

ρ increase the prospect that every party wins a single seat. In turn, this shifts incumbents' concern *away* from voters and *towards* the party leadership. When $\rho \geq \hat{\rho}$, even representatives that share their constituent's preferences for positive surplus projects toe the party line in contexts where the project conveys a negative net surplus to the district.

Polarization. We next compare the consequences of a more polarized electorate for legislative cohesion and party discipline under single- and multi-member districts.

Proposition 4. As the electorate becomes more polarized i.e., ϕ decreases, legislative cohesion in multi-member districts *increases*, i.e., both $\overline{\theta}^M$ and $\underline{\theta}^M$ decrease. This effect becomes more powerful as the proportionality of the electoral rule ρ increases. Under single-member districts, legislative cohesion is unaffected by changes in polarization.

Citizens in a more polarized electorate are relatively more likely to cast their ballots based on partisan affiliation, rather than perceptions of legislators' alignment, diminishing the value to representatives from cultivating a personal reputation with voters,

Party leaders, however, still care about the loyalty of their members, who will be called upon in the subsequent legislative cycle to vote on its agenda. And, a more ideologically dispersed electorate is less likely to vote in sufficient numbers for either ticket to guarantee that it wins two seats. Thus, lower values of ϕ raise the prospect that the Incumbent ticket wins only one seat, and therefore raises the value of being ranked first on the party list. This shifts representatives' concerns towards cultivating a favorable reputation with the party leadership, encouraging both an aligned and a mis-aligned representative to support the leadership in the first-period vote.

In multi-member districts, more generally, polarization in the electorate augments the leadership's control of individual electoral incentives to an even greater extent as the proportionality of the electoral rule, ρ, increases. In single-member districts, however, a greater dispersion of individual voters' party attachments does not affect their electoral incentives, so long as on average those affiliations (i.e., the mean of σ^j) remains centered on zero.

Legislative Accountability. We now consider two normative comparisons between single- and multi-member districts. The first comparison is with respect to first-period payoffs for the voter, which we call *legislative accountability.* This comparison elaborates how distortions arising from incentives to pander to voters under single-member districts and to party leaders under multi-member districts generate a ranking of electoral systems.

Proposition 5. If the conflict between voters and mis-aligned politicians $b > 0$ is not too small, there exists $\rho^*(\phi, b, \mu) \geq 0$, *increasing* in ϕ and *decreasing* in b, such that legislative accountability is higher in single-member districts than multi-member districts, if and only if $\rho \geq \rho^*$.

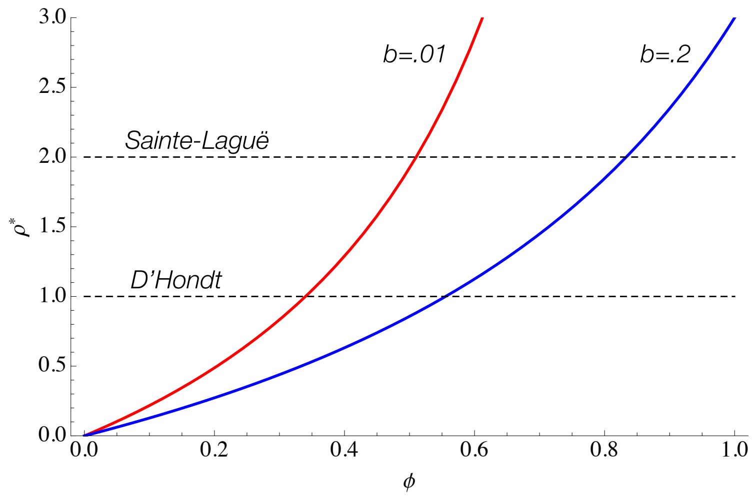

Figure 3: Illustrating the threshold ρ^*, below which MMD yields higher accountability than SMD, for different levels of polarization ϕ, and mis-alignment, b. Higher values of ϕ indicate *less* polarized societies. Parameters: $\mu = .5$, $\chi_1 = \chi_2$, $q(\mathsf{n}, \mathsf{n}) = 0$, $\psi RS = 1$. This yields $\rho^* = \frac{2\phi(1-2b)}{1-\phi(1-2b)}$. The red line is $b = .01$, and the blue line is $b = .2$.

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The shift from single- to multi-member districts generates two distinct effects. The first is that voters cast their ballots for a *team* of politicians, rather than a *single* politician. This implies that any given politician's prospect of re-election depends directly on the collective reputation of the team; it therefore forces incumbents to take into account the damage they do to other politicians' reputations when they try to cultivate their own personal reputation with voters.

The second difference, however, is that the control of electoral incentives partially shifts from voters to political parties. This shift in electoral control weighs more heavily on in individual legislators as polarization increases and as the proportionality of the electoral system increases. When ρ is sufficiently large, all incumbents behave like pusillanimous party puppets, supporting negative surplus projects in order to cultivate a personal reputation with the party leadership. This conveys a harm to voters that may outweigh the harm from excessive obstructionin single-member contexts. If party reputations are initially very high, i.e., μ is sufficiently large, single-member districts are superior for *all* $\rho \geq 0$. The reason is that when μ is large unilateral dissent (a reputation-boosting strategy in single member districts) carries a damage to the party's collective reputation, thus creating ad additional force—independent of the leadership's list choice—that pushes incumbents to coast on their initial reputation by "hiding" behind their party' agenda at the expense of their local voters.

Figure 3 illustrates the threshold ρ^*, and how it varies with primitives. Higher levels of partisan polarization (i.e., *lower* ϕ) make voting behavior less responsive to beliefs about alignment, and thus provide less electoral discipline to incumbent politicians. Higher levels of intrinsic mis-alignment, b, make mis-aligned politicians more prone to voting in favor of projects that harm their districts. To the extent that personal reputation-building incentives work against this behavior, SMD becomes relatively more attractive as a means to counter-balance this incentive. In the figure, we highlight the *D'Hondt* ($\rho = 1$) and *Sainte-Laguë* ($\rho = 2$) systems, to show that the welfare comparison is relevant in the context of real-world variants of multi-member districts.

Our finding that MMD can be superior for relatively low levels of electoral system proportionality, i.e., small ρ, has a striking parallel with Hix and Carey's argument that proportional representation combined with low district magnitude is the 'electoral sweet spot' (Carey and Hix (2011)). They compare low-magnitude PR favorably with single-member districts on the basis that it better trades off representation of heterogeneous preferences in national parliaments (via proportional representation) while maintaining relatively close individual accountability of legislators to voters (via low-magnitude districts). We, instead, emphasize the trade-off between *individual* and *collective* accountability, that may be calibrated most effectively by multi-member districts with low—but positive—proportionality.

Political Selection. Our next result highlights a novel mechanism through which single-member districts *always* induce better political selection than multi-member districts.

Proposition 6. A voter's expected date-two payoff is strictly higher under single-member districts than multi-member districts.

There are three distinct channels through which political selection may vary across systems.

First, differences in ballot structure may facilitate or inhibit the flexibility of voters to remove mis-aligned politicians and retain aligned politicians. Multi-member districts necessarily handicap this flexibility, since a vote in favor of any politician necessarily raises the prospect that any politician on the slate is re-elected.[9] And, list electoral rules partially hand control over the ballot order to party leaderships, empowering them to promote the re-election prospects of mis-aligned representatives. This constitutes a *mechanical* effect that holds regardless of voting strategies.

Second, re-election oriented politicians anticipate how their date-one behavior will affect their prospect of re-election under each ballot structure. To the extent that different ballots induce different incentives to cultivate personal reputation, and thus change the information

[9]We emphasize that this is also true under all forms of flexible list PR, since a vote for one politician still increases the prospect that the remaining politicians are elected. While closed list proportional representation generates this effect most powerfully, it is solely a difference of magnitude.

available to voters, different ballots will also foment different degrees of informativeness in the inference voters make, and which will drive their retention decisions. This is a *strategic* effect derived from the previous point.

Third, voters' decisions at the polls are driven both by their beliefs about the alignment of their representatives and by their partisan dispositions, as reflected in the preference shock σ^j. The relative emphasis placed on each of these components depends partly on whether voters believe that their vote is relatively efficacious in determining the alignment of their date-two politicians.

Under single-member districts, a voter casts her ballot as if she were decisive for the retention or replacement of her representative. That is, she casts her ballot as if she were decisive for raising the prospect of an aligned date-two representative from μ_O to μ. In essence, single-member districts give voters full control over the alignment of their date-two representative.

Consider, by contrast, a voter's decision under multi-member districts. She recognizes that her vote is decisive in the election for one of *two* possible events: raising the number of politicians that are re-elected at date-two from zero to one, or instead from one to two. Which of these two events realizes depends on the choices made by other voters. As such, no single voter can be decisive for the alignment of *both* date-two politicians. Recognizing that her vote is relatively less consequential for the alignment of date-two politicians, voters endogenous place relatively greater emphasis on partisan affiliation. For this reason, selection is necessary worse under multi-member districts. This is a *strategic* i.e., behavioral consequence of the mechanical effect identified as our first channel.

Our analytical approximation (i.e., taking $\kappa \to \infty$) implies that the third channel is the driving force behind Proposition 6. It therefore highlights a novel mechanism through which ballot structure may induce political failure, distinct even from the machinations of party leaderships and the reputation-cultivation of incumbent politicians. In addition to each of these features, our framework highlights that multi-member districts make individual voters relatively less decisive for the total alignment of their date two representatives. In turn, their voting behavior becomes relatively less responsive to changes in beliefs about the alignment of individual representatives, reverting in favor of partisanship and other unrelated factors. This shift necessarily weakens selection—note that our result is true for *all* primitives, including the electoral quota $\rho \geq 0$.

6. Concluding Comments

In this paper, we propose a tractable framework to study legislative cohesion, political selection, and the quality of legislative representation under a wide spectrum of electoral institutions—both single-member, and multi-member.

Our results highlight how politicians' incentives to cultivate *personal* versus *collective* reputation may lead either to excessive obstructionism or instead rubber-stamping of policy agendas. In turn, we show how these individual incentives translate into superior accountability and selection under single- versus multi-member district contexts. We further derive empirical predictions about how legislative cohesion varies with primitives such as polarization.

Much work lies ahead. In a related project, we extend the framework developed in this paper to consider open-list proportional representation systems. We show that legislative cohesion under these electoral rules can fully span the range between closed lists and single-member district settings, providing a theoretical explanation for ambivalent empirical findings reported by Sieberer (2006), who found that countries using open-list proportional representation achieved both the *highest* and *lowest* measures of party unity in legislative voting, in a sample of Western European countries. Extension to flexible-list systems, in which parties and voters share a degree of control over the ordering of incumbent politicians is ongoing. This is particularly important since the vast majority of real-world multi-member contexts that do not use closed lists tend to employ some form of flexible-list proportional representation.

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Appendix

Proof of Proposition 1

Analytic Approximation. First, we can write

$$V_I^S(\mathsf{t}) - V_O^S(\mathsf{t}) = (\hat{\mu}_i(\mathsf{t}) - \mu_o)S + \Delta(\mathsf{t}, \kappa)$$

where

$$\begin{aligned}\Delta(\mathsf{t}, \kappa) \equiv &\mathbb{E}[-b \leq \theta \leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}), \hat{\mu}_j(\mathsf{t}), p_j(\mathsf{t})) + \mathbb{E}[0 \leq \theta \leq b]Q_{[0,b]}(\mu_O, p_j(\mathsf{t})) + \\ &- \mathbb{E}[-b \leq \theta \leq 0]Q_{[-b,0]}(\hat{\mu}_j(\mathsf{t}), p_j(\mathsf{t})) - \mathbb{E}[0 \leq \theta \leq b]Q_{[0,b]}(\mu_O, \mu_O, p_j(\mathsf{t})) \\ =&\mathcal{O}(1/k).\end{aligned}$$

Recall that both $\hat{\mu}_i(\mathsf{t})$ and $\Delta(\mathsf{t}, \kappa)$ depend on both $\underline{\theta}$ and $\overline{\theta}$. Moreover, $\Delta(\mathsf{t}, \kappa) \in \left[-\frac{b^2}{k}, \frac{b^2}{k}\right]$.

A pair $(\underline{\theta}, \overline{\theta})$ is a symmetric equilibrium under SMD if and only if:

$$\Phi(\overline{\theta}, \underline{\theta}) = \overline{\theta} - \frac{RS\psi}{\chi_2}\left(1 - \hat{\mu}_i(\mathsf{y},\mathsf{y}) + \frac{\Delta(\mathsf{n},\mathsf{y},\kappa) - \Delta(\mathsf{y},\mathsf{y},\kappa)}{S}\right) = 0, \quad (25)$$

$$\Psi(\overline{\theta}, \underline{\theta}) = \underline{\theta} + b - \frac{RS\psi}{(1-\mu)\chi_2 + \mu\chi_1}\left[\begin{array}{c} (1-\mu)\left(1 - \hat{\mu}_i(\mathsf{y},\mathsf{y}) + \frac{\Delta(\mathsf{n},\mathsf{y},\kappa)-\Delta(\mathsf{y},\mathsf{y},\kappa)}{S}\right) + \\ +\mu\left(\hat{\mu}_i(\mathsf{n},\mathsf{n}) + \frac{\Delta(\mathsf{n},\mathsf{n},\kappa)-\Delta(\mathsf{y},\mathsf{n},\kappa)}{S}\right) \end{array}\right] = 0. \quad (26)$$

Let

$$[\underline{\theta}_{min}, \underline{\theta}_{max}] \equiv \left[-\frac{R\psi}{(1-\mu)\chi_2 + \mu\chi_1}\frac{b^2}{k} - b, \frac{R\psi}{(1-\mu)\chi_2 + \mu\chi_1}\left(S + \frac{b^2}{k}\right) - b\right]$$
$$[\overline{\theta}_{min}, \overline{\theta}_{max}] \equiv \left[-\frac{R\psi}{\chi_2}\frac{b^2}{k}, \frac{R\psi}{\chi_2}\left(S + \frac{b^2}{k}\right) - b\right]$$

Claim 1. If the uncertainty about the value of the project κ is sufficiently large, then
(i) for each $\underline{\theta} \in [\underline{\theta}_{min}, \underline{\theta}_{max}]$, there exists a unique $\overline{\theta}'(\underline{\theta}) \in [\overline{\theta}_{min}, \overline{\theta}_{max}]$ such that $\Phi(\overline{\theta}', \underline{\theta}) = 0$.
(ii) for each $\overline{\theta} \in [\overline{\theta}_{min}, \overline{\theta}_{max}]$, there exists a unique $\underline{\theta}'(\overline{\theta}) \in [\underline{\theta}_{min}, \underline{\theta}_{max}]$ such that $\Psi(\overline{\theta}, \underline{\theta}') = 0$.

Proof. First, notice that since $\Delta(\mathsf{t}, \kappa) = \mathcal{O}(1/k)$, we have that there exists $\hat{\kappa}_1$ such that when $\kappa \geq \hat{\kappa}_1$ the sign of the derivative $\Phi_1(\overline{\theta}, \underline{\theta})$ is the same as the sign of

$$\frac{\partial}{\partial\overline{\theta}}\left\{\overline{\theta} - \frac{RS\psi}{\chi_2}(1 - \hat{\mu}_i(\mathsf{y},\mathsf{y}))\right\} = 1 - \frac{RS\psi}{\chi_2}\frac{\mu(1-\mu)^2}{\left[1 + \frac{F(\overline{\theta})-F(\underline{\theta})}{1-F(\overline{\theta})}(1-\mu)^2\right]^2}\frac{f(\overline{\theta})}{1-F(\overline{\theta})}\frac{1-F(\underline{\theta})}{1-F(\overline{\theta})}$$

(notice that the expression above, derived from expression (5) does not vanish as κ approaches infinity). Under the supposition $\theta \sim U[-\kappa, \kappa]$, it is sufficient to provide conditions such that:

$$\frac{RS\psi}{\chi_2}\frac{f(\overline{\theta})}{1-F(\overline{\theta})}\frac{1-F(\underline{\theta})}{1-F(\overline{\theta})} < 1 \Leftarrow \frac{RS\psi}{\chi_2}\frac{\kappa - \underline{\theta}}{(\kappa - \overline{\theta})^2} < 1 \quad (27)$$

Since for all interior $(\underline{\theta}, \underline{\theta})$ (that is $\kappa > \max\{\overline{\theta}_{max}, \underline{\theta}_{max}\}$), $\frac{\kappa-\underline{\theta}}{(\kappa-\overline{\theta})^2}$ is strictly decreasing in κ, and $\lim_{\kappa\to\infty}\frac{\kappa-\underline{\theta}}{(\kappa-\overline{\theta})^2} = 0$, we conclude that there exists a finite κ_1 such that $\kappa \geq \max\{\hat{\kappa}_1, \kappa_1\} \Rightarrow \Phi_1(\overline{\theta}, \underline{\theta}) > 0$. Since $\Phi(\cdot, \underline{\theta})$ is continuous, $\Phi(\overline{\theta}_{min}, \underline{\theta}) < 0$ and $\Phi(\overline{\theta}_{max}, \underline{\theta}) < 0$ $\forall \underline{\theta} \in [\underline{\theta}_{min}, \underline{\theta}_{max}]$, existence of a unique solution $\overline{\theta}'$ solving $\Phi(\overline{\theta}', \underline{\theta}) = 0$ follows. By a similar reasoning, using the expression of $\hat{\mu}_i(\mathsf{y},\mathsf{y})$, and $\hat{\mu}_i(\mathsf{n},\mathsf{n})$, we establish that there exists $(\hat{\kappa}_2\ \kappa_2)$ such that (i) when $\kappa geq \hat{\kappa}_2$, the derivative $\Psi_2(\overline{\theta}, \cdot)$ has the same sign of

$$\frac{\partial}{\partial \underline{\theta}} \left\{ \underline{\theta} + b - \frac{RS\psi}{(1-\mu)\chi_2 + \mu\chi_1} \left[\begin{array}{c} (1-\mu)\,(1-\hat{\mu}_i(\mathsf{y},\mathsf{y})) + \\ +\mu\,(\hat{\mu}_i(\mathsf{n},\mathsf{n})) \end{array} \right] \right\}$$

and (ii) when $\kappa \geq \max\{\hat{\kappa}_2, \kappa_2\}$, $\Psi(\overline{\theta}, \cdot)$ is strictly increasing with a unique zero. □

Claim 2. If κ is large enough, $\overline{\theta}'(\underline{\theta})$ is strictly *decreasing* in $\underline{\theta}$, and $\underline{\theta}'(\overline{\theta})$ is strictly *increasing* in $\overline{\theta}$.

Proof. We show that $\overline{\theta}'(\underline{\theta})$ is strictly *decreasing*. By the same reasoning as in the previous claim, we can establish that for κ large enough, the sign of the derivative $\Phi_2(\overline{\theta}, \underline{\theta})$ is the same as the sign of

$$\frac{\partial}{\partial \underline{\theta}} \left\{ \overline{\theta} - \frac{RS\psi}{\chi_2}\,(1-\hat{\mu}_i(\mathsf{y},\mathsf{y})) \right\} \propto \frac{\partial \hat{\mu}_i(\mathsf{y},\mathsf{y})}{\partial \underline{\theta}} \propto f(\underline{\theta})(F(\underline{\theta})-1) < 0.$$

Since $\Phi_1(\overline{\theta}, \underline{\theta}) > 0$, a higher $\underline{\theta}$ shifts the zero of $\Phi(\cdot, \underline{\theta})$ to the left. Likewise, we observe that for κ large enough, the sign of $\Psi_1(\overline{\theta}, \underline{\theta})$ is the same as the sign of

$$\frac{\partial}{\partial \overline{\theta}} \left\{ \underline{\theta} + b - \frac{RS\psi}{(1-\mu)\chi_2 + \mu\chi_1} \left[\begin{array}{c} (1-\mu)\,(1-\hat{\mu}_i(\mathsf{y},\mathsf{y})) + \\ +\mu\,(\hat{\mu}_i(\mathsf{n},\mathsf{n})) \end{array} \right] \right\} \propto \mu \frac{\partial \hat{\mu}_i(\mathsf{n},\mathsf{n})}{\partial \overline{\theta}} - (1-\mu)\frac{\partial \hat{\mu}_i(\mathsf{y},\mathsf{y})}{\partial \overline{\theta}}$$

Tedious but straightforward algebra yields $\frac{\partial \hat{\mu}_i(\mathsf{y},\mathsf{y})}{\partial \overline{\theta}} < 0$ and $\frac{\partial \hat{\mu}_i(\mathsf{n},\mathsf{n})}{\partial \overline{\theta}} > 0$ (intuitively, increasing the propensity of an aligned type to vote against raises $\hat{\mu}_i(\mathsf{y},\mathsf{y})$ and reduces $\hat{\mu}_i(\mathsf{y},\mathsf{y})$). Since $\Psi_2(\overline{\theta}, \underline{\theta}) > 0$, a higher $\overline{\theta}$ shifts the zero of $\Psi(\overline{\theta}, \cdot)$ to the right. □

Claim 3. If $b > 0$ is sufficiently large, a symmetric equilibrium exists and every symmetric equilibrium satisfies $\underline{\theta}^S < \overline{\theta}^S$. If, in addition, κ is large enough, a unique symmetric equilibrium exists.

Proof. Since $[\underline{\theta}_{min}, \underline{\theta}_{max}]$ and $[\overline{\theta}_{min}, \overline{\theta}_{max}]$ are both convex and compact, their product set is convex and compact. The equilibrium correspondence $G : [\underline{\theta}_{min}, \underline{\theta}_{max}] \times [\overline{\theta}_{min}, \overline{\theta}_{max}] \to [\underline{\theta}_{min}, \underline{\theta}_{max}] \times [\overline{\theta}_{min}, \overline{\theta}_{max}]$ represented by equations (25) and (26) is continuous, and thus has at least one fixed point. For $b > 0$ sufficiently large, any fixed point satisfies $\underline{\theta}^S < \overline{\theta}^S$. Uniqueness when κ is large enough follows from the previous Claims. □

Claim 4. For any $\epsilon > 0$, there exists $(\underline{\kappa}(\epsilon), \overline{\kappa}(\epsilon))$ such that $\kappa > \max\{\overline{\kappa}(\epsilon), \underline{k}(epsilon)\}$ implies:

$$0 < \overline{\theta}^S - \frac{RS\psi}{\chi_2}(1-\mu) < \epsilon \tag{28}$$

$$0 < \underline{\theta}^S - \frac{\psi RS}{(1-\mu)\chi_2 + \mu\chi_1}(1-2\mu(1-\mu)) - b < \epsilon \tag{29}$$

Proof. We have:

$$\begin{aligned}
\overline{\theta}^S - \frac{RS\psi}{\chi_2}(1-\mu) &= \frac{RS\psi}{\chi_2}(\mu - \hat{\mu}_i(1,1)) &(30)\\
&= \frac{RS\psi}{\chi_2}\mu\left(1 - \frac{1}{1 + \frac{F(\overline{\theta})-F(\underline{\theta})}{1-F(\overline{\theta})}(1-\mu)^2}\right) &(31)\\
&= \frac{RS\psi}{\chi_2}\mu\left(\frac{\phi(\kappa)(1-\mu)^2}{1+\phi(\kappa)(1-\mu)^2}\right), &(32)
\end{aligned}$$

where:

$$\phi(\kappa) = \frac{F(\overline{\theta}) - F(\underline{\theta})}{1 - F(\overline{\theta})} < \frac{F\left(\overline{\theta}_{max}\right) - F\left(\underline{\theta}_{min}\right)}{1 - F\left(\overline{\theta}_{max}\right)} = \frac{\overline{\theta}_{max} - \underline{\theta}_{min}}{\kappa - \overline{\theta}_{max}}. \tag{33}$$

The final term on the RHS is strictly decreasing in κ, hence there is a unique $\overline{\kappa}(\epsilon)$ such that $\epsilon = \frac{RS\psi}{\chi_2}\mu\left(\frac{\phi(\overline{\kappa}(\epsilon))(1-\mu)^2}{1+\phi(\overline{\kappa}(\epsilon))(1-\mu)^2}\right)$. The proof for $\underline{\kappa}(\epsilon)$ proceeds in the same fashion. □

We conclude from the above that if $b > 0$ is not too small, (1) for κ large enough, there exists a unique symmetric equilibrium satisfying $\overline{\theta}^S > \underline{\theta}^S$ and (2) that by appropriate choice of support $[-\kappa, \kappa]$, the unique symmetric equilibrium can be approximated arbitrarily well by:

$$\overline{\theta}^S = \frac{RS\psi}{\chi_2}(1-\mu) \tag{34}$$

$$\underline{\theta}^S = \frac{\psi RS}{(1-\mu)\chi_2 + \mu\chi_1}(1-2\mu(1-\mu)) - b. \tag{35}$$

Obtaining the thresholds Using a slightly generalized version of expressions (8) and (9), we can write the payoff from retaining incumbent $i \in \{A, B\}$ as

$$\begin{aligned}
V_I^S(\mathsf{t}) =& \mu_i(\mathsf{t})S + \mathbb{E}[\theta \leq -b]q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \geq b]q(\mathsf{y},\mathsf{y}) + \\
&+ \mathbb{E}[-b \leq \theta \leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}), \hat{\mu}_j(\mathsf{t}), p_j(\mathsf{t})) + \mathbb{E}[0 \leq \theta \leq b]Q_{[0,b]}(\mu_O, p_j(\mathsf{t}))
\end{aligned}$$

where $Q_{[x',x'']}(\cdot, p_j(\mathsf{t})) \in [0,1]$ is the probability of passage of a policy agenda whose value is in $[x', x'']$ given that the other legislator is retained with probability $p_j(\mathsf{t})$, as a function of voters' beliefs about representatives' types. When $\theta \in [0, b]$, there is no uncertainty about the behavior of an incumbent, so the only relevant belief is the one about the opposition challenger; when $\theta \in [-b, 0]$, there is no uncertainty about the behavior of an opposition challenger, so the relevant beliefs are the ones about incumbents i and j. Similarly, we have

$$\begin{aligned} V_O^S(\mathsf{t}) =& \mu_O S + \mathbb{E}[\theta \leq -b] q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \geq b] q(\mathsf{y},\mathsf{y}) + \\ & + \mathbb{E}[-b \leq \theta \leq 0] Q_{[-b,0]}(\hat{\mu}_j(\mathsf{t}), p_j(\mathsf{t})) + \mathbb{E}[0 \leq \theta \leq b] Q_{[0,b]}(\mu_O, \mu_O, p_j(\mathsf{t})) \end{aligned}$$

First, continuity of beliefs in $\underline{\theta}(\kappa)$ and $\overline{\theta}^S(\kappa)$ allows to establish existence using Brower's theorem.[10] As κ approaches infinity, we have that

$$\lim_{\kappa\to\infty} \mathbb{E}[-b \leq \theta \leq 0] = \lim_{\kappa\to\infty} \int_{-b}^{0} \frac{z}{2\kappa} dz = 0 = \lim_{\kappa\to\infty} \mathbb{E}[0 \leq \theta \leq b]$$

which implies that $V_I^S(\mathsf{t}) - V_O^S(\mathsf{t}) \to_{\kappa\to\infty} S(\mu_i(\mathsf{t}) - \mu_O)$, and by equation 7, $p_i^S(\mathsf{t}) = \frac{1}{2} + \psi S[\mu_i(\mathsf{t}) - \mu_O]$. Finally, by inspection of expression (5) and the analogous expression for $\hat{\mu}_i(\mathsf{n},\mathsf{n})$, we also have

$$\lim_{\kappa\to\infty} \hat{\mu}(\mathsf{y},\mathsf{y}) = \lim_{\kappa\to\infty} \hat{\mu}(\mathsf{n},\mathsf{n}) = \mu,$$

Which implies that $p_i^S(\mathsf{y},\mathsf{y}) - p_i^S(\mathsf{n},\mathsf{y}) = \psi RS[\mu - 1]$ and $p_i^S(\mathsf{y},\mathsf{n}) - p_i^S(\mathsf{n},\mathsf{n}) = \psi RS[-\mu]$. Plugging these values into (4) and (6) yields the values of $\overline{\theta}^S$ and $\underline{\theta}^S$. Uniqueness for finite κ follows from continuity.

Proof of Lemma 2

The proof proceeds in two steps. First, we derive $V_I^M(\mathsf{t}, \{ij\})$ and $V_O^M(\mathsf{t}, \{ij\})$. Then we show that, for each t, $V_I^M(\mathsf{t}, \{ij\}) - V_O^M(\mathsf{t}, \{ij\})$ is independent of the list order.

Step 1. In the single-member district case, the difference $V_I^S(\mathsf{t}) - V_O^M(\mathsf{t})$ is driven by the unique pivotal event that each voter needs to consider: when her district's incumbent and his challenger have the same vote share ($\Pi = \frac{1}{2}$). In that case, she computes the expected value of retaining her incumbent and compares with the value of replacing him. Under multi-member districts, there are two possible pivotal events: (i) the incumbent ticket gets just enough votes to secure one seat ($\Pi = \frac{1}{2+\rho}$) which happens when

$$\xi = \overline{\xi}_1 = V_I^M(\mathsf{t}, \{ij\}) - V_O^M(\mathsf{t}, \{ij\}) + \frac{1}{2\phi} \frac{\rho}{\rho + 2},$$

[10]To ensure that the mapping is bounded and compact, we can take $(\underline{\theta}, \overline{\theta}) \in \left[-\frac{R}{\chi_2}, \frac{R}{\chi_2}\right] \times \left[-b - \frac{R}{\mu\chi_1+(1-\mu)\chi_2}, -b + \frac{R}{\mu\chi_1+(1-\mu)\chi_2}\right]$, which is implied by (4) and (6).

and (ii) the incumbent ticket gets just enough votes to secure two seats ($\Pi = \frac{\rho+1}{\rho+2}$), which happens when

$$\xi = \overline{\xi}_2 = V_I^M(\mathsf{t}, \{ij\}) - V_O^M(\mathsf{t}, \{ij\}) - \frac{1}{2\phi}\frac{\rho}{\rho+2},$$

As a result, $V_I^M(\mathsf{t}, \{ij\}) - V_O^M(\mathsf{t}, \{ij\})$ can be written as the simple average of the payoff difference between (i) retention or replacement of the top-listed incumbent i when $\xi = \overline{\xi}_1$ and (ii) retention or replacement of the bottom listed j when $\xi = \overline{\xi}_2$. As a result, proceeding as in the proof of Proposition 1, we obtain:[11]

$$\begin{aligned}
&V_I^M(\mathsf{t}, \{ij\}) = \\
&= \frac{Pr(\xi = \overline{\xi}_2)}{Pr(\xi = \overline{\xi}_2) + Pr(\xi = \overline{\xi}_1)} \left\{ \begin{array}{c} \frac{\mu_i(\mathsf{t})+\mu_j(\mathsf{t})}{2} S + \mathbb{E}[\theta \le -b] q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \ge b] q(\mathsf{y},\mathsf{y}) \\ +\mathbb{E}[-b \le \theta \le 0] Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}), \hat{\mu}_j(\mathsf{t})) + \mathbb{E}[0 \le \theta \le b] q(\mathsf{y},\mathsf{y}) \end{array} \right\} \\
&+ \frac{Pr(\xi = \overline{\xi}_1)}{Pr(\xi = \overline{\xi}_2) + Pr(\xi = \overline{\xi}_1)} \left\{ \begin{array}{c} \frac{\mu_i(\mathsf{t})+\mu_O}{2} S + \mathbb{E}[\theta \le -b] q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \ge b] q(\mathsf{y},\mathsf{y}) \\ +\mathbb{E}[-b \le \theta \le 0] Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t})) + \mathbb{E}[0 \le \theta \le b] Q_{[0,b]}(\mu_O) \end{array} \right\}
\end{aligned}$$

where $Q_{[x',x'']}(\cdot)$ is the probability of passage of a policy agenda whose value is in $[x', x'']$ as a function of voters' beliefs about representatives' types. Again, when $\theta \in [-b, 0]$ the only relevant beliefs are about the retained incumbent representatives (both under the pivotal event $\xi = \overline{\xi}_1$, only i under the pivotal event $\xi = \overline{\xi}_2$); when $\theta \in [0, b]$ the only relevant beliefs are about the elected opposition challengers. Similarly, we have

$$\begin{aligned}
&V_O^M(\mathsf{t}, \{ij\}) = \\
&= \frac{Pr(\xi = \overline{\xi}_2)}{Pr(\xi = \overline{\xi}_2) + Pr(\xi = \overline{\xi}_1)} \left\{ \begin{array}{c} \frac{\mu_i(\mathsf{t})+\mu_O}{2} S + \mathbb{E}[\theta \le -b] q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \ge b] q(\mathsf{y},\mathsf{y}) \\ +\mathbb{E}[-b \le \theta \le 0] Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t})) + \mathbb{E}[0 \le \theta \le b] Q_{[0,b]}(\mu_O) \end{array} \right\} \\
&+ \frac{Pr(\xi = \overline{\xi}_1)}{Pr(\xi = \overline{\xi}_2) + Pr(\xi = \overline{\xi}_1)} \left\{ \begin{array}{c} \frac{2\mu_O}{2} S + \mathbb{E}[\theta \le -b] q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \ge b] q(\mathsf{y},\mathsf{y}) \\ +\mathbb{E}[-b \le \theta \le 0] q(\mathsf{n},\mathsf{n}) + \mathbb{E}[0 \le \theta \le b] Q_{[0,b]}(\mu_O, \mu_O) \end{array} \right\}
\end{aligned}$$

[11]Since voters are served by two legislators, to obtain comparable payoffs we scale the non-legislative payoff S by two.

Step 2. Since ξ is drawn from a uniform, $\frac{Pr(\xi=\bar{\xi}_1)}{Pr(\xi=\bar{\xi}_2)+Pr(\xi=\bar{\xi}_1)} = \frac{Pr(\xi=\bar{\xi}_2)}{Pr(\xi=\bar{\xi}_2)+Pr(\xi=\bar{\xi}_1)} = \frac{1}{2}$. Hence, we obtain

$$
\begin{aligned}
&V_I^M(\mathsf{t},\{ij\}) - V_O^M(\mathsf{t},\{ij\}) = \\
&= \frac{1}{2}\left\{ \begin{array}{c} \frac{\mu_j(\mathsf{t})-\mu_O}{2}S + \mathbb{E}[-b \le \theta \le 0][Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}),\hat{\mu}_j(\mathsf{t})) - Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}))] \\ +\mathbb{E}[0 \le \theta \le b][q(\mathsf{y},\mathsf{y}) - Q_{[0,b]}(\mu_O)] \end{array} \right\} \\
&+ \frac{1}{2}\left\{ \begin{array}{c} \frac{\mu_i(\mathsf{t})-\mu_O}{2}S + \mathbb{E}[-b \le \theta \le 0][Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t})) - q(\mathsf{n},\mathsf{n})) \\ +\mathbb{E}[0 \le \theta \le b][Q_{[0,b]}(\mu_O) - Q_{[0,b]}(\mu_O,\mu_O)] \end{array} \right\} \\
&= \frac{\mu_i(\mathsf{t}) + \mu_j(\mathsf{t}) - 2\mu_O}{4}S \\
&+ \frac{\mathbb{E}[-b \le \theta \le 0]}{2}\Big[Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}),\hat{\mu}_j(\mathsf{t})) - q(\mathsf{n},\mathsf{n})\Big] + \frac{\mathbb{E}[0 \le \theta \le b]}{2}\Big[q(\mathsf{y},\mathsf{y}) - Q_{[0,b]}(\mu_O,\mu_O)\Big]
\end{aligned}
$$

The last expression implies that the electoral attractiveness of the ticket is independent of the list order:

$$V_I^M(\mathsf{t},\{ij\}) - V_O^M(\mathsf{t},\{ij\}) = V_I^M(\mathsf{t},\{ji\}) - V_O^M(\mathsf{t},\{ji\})$$

which implies that $\forall \mathsf{t} \in \{\mathsf{y},\mathsf{n}\}^2$, $p_j(\mathsf{t},\{ij\}) = p_i(\mathsf{t},\{ji\}) = p_2(\mathsf{t})$ and $p_i(\mathsf{t},\{ij\}) = p_j(\mathsf{t},\{ji\}) = p_1(\mathsf{t})$. In equilibrium, the top-listed candidates is more likely to be elected: by (15) and (16), we have:

$$p_1(\mathsf{t}) - p_2(\mathsf{t}) = \frac{\psi}{\phi}\frac{\rho}{\rho+2}.$$

We conclude by showing that the leadership's expected second-period payoff is higher when the candidate with the lower perceived constituency alignment after the first period vote is given electoral priority. To see that, suppose $\hat{\mu}_i(\mathsf{t}) < \hat{\mu}_j(\mathsf{t})$. The relative value of giving priority to i is given by the difference in the probability of passage under the two lists $\{ij\}$ and $\{ji\}$. Since when $|\theta| > b$ both types vote in the same way and the total expected number of seats $p_1(\mathsf{t}) + 2p_2(\mathsf{t})$ does not depend on the list order, this difference can be written as (omitting the dependence on the vote tally, for visual clarity)

$$
\begin{aligned}
&Q_{[-b,0]}(\hat{\mu}_i,\hat{\mu}_j)p_2 + Q_{[-b,0]}(\hat{\mu}_i)(p_1 - p_2) + Q_{[0,b]}(\mu_O)(p_1 - p_2) + Q_{[0,b]}(\mu_O,\mu_O)(1 - p_1) \\
&- Q_{[-b,0]}(\hat{\mu}_i,\hat{\mu}_j)p_2 + Q_{[-b,0]}(\hat{\mu}_j)(p_1 - p_2) + Q_{[0,b]}(\mu_O)(p_1 - p_2) + Q_{[0,b]}(\mu_O,\mu_O)(1 - p_1) \\
&= [Q_{[-b,0]}(\hat{\mu}_i) - Q_{[-b,0]}(\hat{\mu}_j)](p_1 - p_2) \\
&= [\hat{\mu}_j - \hat{\mu}_i]\chi_1(p_1 - p_2) > 0.
\end{aligned}
$$

This completes the proof.

Proof of Proposition 2. Combining the proof of Lemma 2 with equations (15) and (16), we have that, as κ approaches infinity,

$$p_i{}^M(\mathsf{t},\{ij\}) = \frac{1}{2} + \frac{\psi}{2\phi}\frac{\rho}{\rho+2} + \psi\frac{S}{2}\left(\frac{\mu_i(\mathsf{t})+\mu_j(\mathsf{t})}{2} - \mu_O\right)$$
$$p_i{}^M(\mathsf{t},\{ji\}) = \frac{1}{2} - \frac{\psi}{2\phi}\frac{\rho}{\rho+2} + \psi\frac{S}{2}\left(\frac{\mu_i(\mathsf{t})+\mu_j(\mathsf{t})}{2} - \mu_O\right).$$

As a result, expressions (17) and (18) become

$$RS\psi\left[\frac{\mu-1/2}{2} + \frac{1}{2S\phi}\frac{\rho}{\rho+2}\right] + \chi_2\overline{\theta}^M. \tag{36}$$

$$RS\psi\left[\begin{array}{c}(1-\mu)\left(\frac{\mu-1/2}{2}+\frac{1}{2S\phi}\frac{\rho}{\rho+2}\right)\\ +\mu\left(\frac{1/2-\mu}{2}+\frac{1}{2S\phi}\frac{\rho}{\rho+2}\right)\end{array}\right] + \left[\begin{array}{c}(1-\mu)\chi_2\\ +\mu\chi_1\end{array}\right](\underline{\theta}^M + b) \tag{37}$$

Setting them to zero yields the result.

Proof of Corollary 1. We have $\frac{d\overline{\theta}^M}{d\rho} = \frac{\Psi RS}{2\chi_2}\frac{(-1)}{S\phi}\frac{2}{(\rho+2)^2} < 0$, and the corresponding result $\frac{d\underline{\theta}^M}{d\rho} < 0$ is similar.

Proof of Proposition 3. We have:

$$\overline{\theta}^S - \overline{\theta}^M = \frac{\Psi RS}{2\chi_2}\left[2(1-\mu) - \left(\frac{1}{2} - \mu - \frac{1}{S\phi}\frac{\rho}{\rho+2}\right)\right]. \tag{38}$$

It is sufficient to observe that $2(1-\mu) - .5 - \mu = 1.5 - \mu > 0$ for all $\mu \in [0,1]$, so that $\overline{\theta}^S - \overline{\theta}^M > 0$. Similarly:

$$\underline{\theta}^S - \underline{\theta}^M = \frac{\Psi RS}{2[(1-\mu)\chi_2 + \mu\chi_1]}\left[2 - 4\mu(1-\mu) - \left(2(.5-\mu)^2 - \frac{1}{S\phi}\frac{\rho}{2+\rho}\right)\right]. \tag{39}$$

It is sufficient to observe that $2-4\mu(1-\mu)-2(.5-\mu)^2 = 1.5-2\mu(1-\mu) > 0$ for all $\mu \in [0,1]$, so that $\underline{\theta}^S - \underline{\theta}^M > 0$.

Proof of Corollary 2. We have $\overline{\theta}^S > 0$ by inspection. We have $\overline{\theta}^M < 0$ if and only if $.5 - \mu - \frac{1}{S\phi}\frac{\rho}{\rho+2} < 0$. This condition holds for all $\rho \geq 0$ if and only if $\mu > .5$. If, instead, $\mu \leq .5$, we have $\overline{\theta}^M < 0$ if and only if $\rho \geq \frac{2S\phi(.5-\mu)}{1-S\phi(.5-\mu)} = \hat{\rho}(\phi,\mu)$.

Proof of Proposition 4. We have that $\frac{d\overline{\theta}^M}{d\phi} = \frac{\Psi RS}{2\chi_2}\frac{\rho}{\rho+2}\frac{1}{S\phi^2} > 0$, so that *lower* ϕ implies $\overline{\theta}^M$ *decreases*. That the magnitude of this effect increases in ρ is by inspection. The corresponding result for the threshold $\underline{\theta}^M$ is similar.

Proof of Proposition 5. First, the welfare under system $E \in \{S, M\}$ can be written as

$$\begin{aligned}
W^E &= q(\mathsf{y},\mathsf{y}) \int_{\overline{\theta}^E}^{\kappa} \frac{z}{2\kappa} dz + [(1-\mu)^2 q(\mathsf{y},\mathsf{y}) + \mu^2 q(\mathsf{n},\mathsf{n}) + 2\mu(1-\mu) q(\mathsf{y},\mathsf{n})] \int_{\underline{\theta}^E}^{\overline{\theta}^E} \frac{z}{2\kappa} dz \\
&= \left[\begin{array}{c} \mu^2(\chi_1+\chi_2) + 2\mu(1-\mu)\chi_2 \\ q(\mathsf{n},\mathsf{n}) + (1-\mu)^2(\chi_1+\chi_2) + 2\mu(1-\mu)\chi_1 \end{array} \right] \int_{\overline{\theta}^E}^{\kappa} \frac{z}{2\kappa} dz + \\
&\quad + [q(\mathsf{n},\mathsf{n}) + (1-\mu)^2(\chi_1+\chi_2) + 2\mu(1-\mu)\chi_1] \int_{\underline{\theta}^E}^{\overline{\theta}^E} \frac{z}{2\kappa} dz \\
&\propto \frac{\mu^2(\chi_1+\chi_2) + 2\mu(1-\mu)\chi_2}{q(\mathsf{n},\mathsf{n}) + (1-\mu)^2(\chi_1+\chi_2) + 2\mu(1-\mu)\chi_1} \int_{\overline{\theta}^E}^{\kappa} z dz + \int_{\underline{\theta}^E}^{\kappa} z dz
\end{aligned}$$

Let

$$\begin{aligned}
\Gamma &= \Gamma(\mu, \mathbf{q}) = \frac{2\mu(1-\mu)\chi_2 + \mu^2(\chi_2+\chi_1)}{q(\mathsf{n},\mathsf{n}) + (1-\mu)^2(\chi_2+\chi_1) + 2\mu(1-\mu)\chi_1} \left(1 - \mu + \mu \frac{\chi_1}{\chi_2}\right)^2 \\
\varrho &= \varrho(\rho, S, \phi) = \frac{1}{S\phi} \frac{\rho}{2+\rho} \\
M &= M(\mu) = \mu(1-\mu)
\end{aligned}$$

We can rewrite

$$\begin{aligned}
\overline{\theta}^S &= \frac{\psi RS}{\chi_2}(1-\mu) \\
\overline{\theta}^M &= \frac{\psi RS}{\chi_2} \frac{1}{2} (\frac{1}{2} - \mu - \varrho) \\
\underline{\theta}^S &= \frac{\psi RS}{\chi_2} \left(1 - \mu + \mu \frac{\chi_1}{\chi_2}\right)^{-1} (1 - 2M) - b \\
\underline{\theta}^M &= \frac{\psi RS}{\chi_2} \left(1 - \mu + \mu \frac{\chi_1}{\chi_2}\right)^{-1} \frac{1}{2} \left(\frac{1}{2} - 2M - \varrho\right) - b
\end{aligned}$$

After some tedious but straightforward algebra, we obtain that

$$\begin{aligned}W^M - W^S &\propto \Gamma\left(1-\mu+\mu\frac{\chi_1}{\chi_2}\right)^{-2}[(\overline{\theta}^S)^2 - (\overline{\theta}^M)^2) + (\underline{\theta}^S)^2 - (\underline{\theta}^M)^2 \\ &\propto \Gamma\left[\frac{15}{4} - \mu(7-3\mu) - \varrho(\varrho - 1 + 2\mu)\right] \\ &+ \left(\frac{3}{2} + \varrho - 2M\right)\left(\frac{5}{2} - 6M - \frac{4b}{\frac{\Psi RS}{\chi_2}(1-\mu+\mu\frac{\chi_1}{\chi_2})^{-1}} - \varrho\right) \\ &= \Delta(\varrho).\end{aligned}$$

Since $\Delta''(\varrho) = -2(1+\Gamma) < 0$, $\Delta(\varrho)$ is strictly concave. Moreover:

$$\Delta'(0) = 1 - 4\tilde{b} - 4\mu(1-\mu) + \Gamma(1-2\mu). \tag{40}$$

where $\tilde{b} = \frac{b}{\frac{\Psi RS}{\chi_2}(1-\mu+\mu\frac{\chi_1}{\chi_2})^{-1}}$. Thus, so long as:

$$\tilde{b} > .25 - \mu(1-\mu) + .25\Gamma(1-2\mu), \tag{41}$$

we have that $\Delta(\varrho)$ strictly decreases in $\varrho \geq 0$, which implies there is at most one $\rho^* \geq 0$ such that if and only if $\rho \geq \rho^*$, $W^M - W^S \leq 0$. Note, further that in order to have $\underline{\theta}_S < \overline{\theta}_S$ and $\underline{\theta}_M < \overline{\theta}_M$, we need

$$\tilde{b} > \max\left\{\mu\left(\mu - \frac{\chi_1}{\chi_2}(1-\mu)\right), \frac{1}{2}\mu\left(2\varrho\left(\frac{\chi_1}{\chi_2} - 1\right) + (2\mu-1)\left(1 + \frac{\chi_1}{\chi_2}\right)\right)\right\} \tag{42}$$

so that the satisfaction of condition (41) is consistent with conditions for existence of the threshold equilibrium, and moreover its uniqueness for κ sufficiently large.

We now show that when (41) holds, ρ^* weakly *decreases* in b and weakly *increases* in ϕ (weakly, since ρ^* may be equal to zero if the root of $\Delta(\varrho)$ is strictly negative). If (41) holds, then $\Delta'(\varrho) < 0$, i.e., $\frac{d\Delta'(\varrho)}{d\rho} < 0$ since ϱ increases in ρ; moreover, $\frac{d\Delta'(\varrho)}{db} < 0$ and $\frac{d\Delta'(\varrho)}{d\phi} > 0$. The comparative statics follow from the Implicit Function Theorem.

Proof of Proposition 6. Under MMD, we compute the expected number of second-period aligned politicians. We use the notational short-hand $p^2(\mathtt{t})$ to denote the probability that both incumbents are retained, $p^1(\mathtt{t})$ to denote the probability that a single incumbent is retained, and $p^0(\mathtt{t}) = 1 - p^2(\mathtt{t}) - p^1(\mathtt{t})$ to denote the probability that neither incumbent is retained. Thus, the expected number of second-period aligned politicians is:

$$
\begin{aligned}
Z^M(\kappa) \equiv& \mu^2(1-F(\overline{\theta}^M))\Big[2p^2(\mathsf{y},\mathsf{y})+p^1(\mathsf{y},\mathsf{y})(1+\mu_O)+p^0(\mathsf{y},\mathsf{y})2\mu_O\Big] \\
&+\mu^2 F(\overline{\theta}^M)\Big[2p^2(\mathsf{n},\mathsf{n})+p^1(\mathsf{n},\mathsf{n})(1+\mu_O)+p^0(\mathsf{n},\mathsf{n})2\mu_O\Big] \\
&+(1-\mu)^2(1-F(\underline{\theta}^M))\Big[p^1(\mathsf{y},\mathsf{y})\mu_O+p^0(\mathsf{y},\mathsf{y})\mu_O\Big] \\
&+(1-\mu)^2 F(\underline{\theta}^M)\Big[p^1(\mathsf{n},\mathsf{n})\mu_O+p^0(\mathsf{n},\mathsf{n})2\mu_O\Big] \\
&+2\mu(1-\mu)(1-F(\overline{\theta}^M))\Big[p^2(\mathsf{y},\mathsf{y})+p^1(\mathsf{y},\mathsf{y})\Big(\frac{2\mu_O+1-\mu_O}{2}+\frac{\mu_O}{2}\Big)+p^0(\mathsf{y},\mathsf{y})2\mu_O\Big] \\
&+2\mu(1-\mu)F(\underline{\theta}^M)\Big[p^2(\mathsf{n},\mathsf{n})+p^1(\mathsf{n},\mathsf{n})\Big(\frac{2\mu_O+1-\mu_O}{2}+\frac{\mu_O}{2}\Big)+p^0(\mathsf{n},\mathsf{n})2\mu_O\Big] \\
&+2\mu(1-\mu)(F(\overline{\theta}^M)-F(\underline{\theta}^M))\Big[p^2(\mathsf{y},\mathsf{n})+p^1(\mathsf{y},\mathsf{n})\mu_O+p^0(\mathsf{y},\mathsf{n})2\mu_O\Big].
\end{aligned} \tag{43}
$$

Noticing that $\lim_{\kappa\to\infty}(F(\overline{\theta}^M)-F(\underline{\theta}^M))=0$, and:

$$
\begin{aligned}
&\lim_{\kappa\to\infty} p^2(\mathsf{y},\mathsf{y}) = \lim_{\kappa\to\infty} p^2(\mathsf{n},\mathsf{n}) = \frac{1}{2}-\frac{\psi}{\phi}\frac{\rho}{2(2+\rho)}+\frac{S\psi}{2}(\mu-\mu_O)\equiv p^2, \\
&\lim_{\kappa\to\infty} p^1(\mathsf{y},\mathsf{y}) = \lim_{\kappa\to\infty} p^1(\mathsf{n},\mathsf{n}) = \frac{\psi}{\phi}\frac{\rho}{2+\rho}\equiv p^1,
\end{aligned} \tag{44}
$$

we obtain:

$$
\lim_{\kappa\to\infty} Z^M(\kappa) = 2p^2\mu+p^1(\mu+\mu_O)+2p^0\mu_C = 2p^2(\mu-\mu_O)+p^1(\mu-\mu_O)+2\mu_O, \tag{45}
$$

where the second equality follows from $p^0=1-p^2-p^1$.

Next, we compute the probability that, in a given district under SMD, the second-period representative is aligned:

$$
\begin{aligned}
Z^S(\kappa) \equiv& \mu^2[(1-F(\overline{\theta}^S))(p(\mathsf{y},\mathsf{y})+(1-p(\mathsf{y},\mathsf{y}))\mu_O)+F(\overline{\theta}^S)(p(\mathsf{n},\mathsf{n})+(1-p(\mathsf{n},\mathsf{n}))\mu_O)] \\
&+(1-\mu)^2[(1-F(\underline{\theta}^S))(1-p(\mathsf{y},\mathsf{y}))\mu_O+F(\underline{\theta}^S)(1-p(\mathsf{n},\mathsf{n}))\mu_O] \\
&+\mu(1-\mu)[(1-F(\overline{\theta}^S))(p(\mathsf{y},\mathsf{y})+(1-p(\mathsf{y},\mathsf{y}))\mu_O)+F(\underline{\theta}^S)(p(\mathsf{n},\mathsf{n})+(1-p(\mathsf{n},\mathsf{n}))\mu_O)] \\
&+\mu(1-\mu)[F(\overline{\theta}^S)-F(\underline{\theta}^S)](p(\mathsf{n},\mathsf{y})+(1-p(\mathsf{n},\mathsf{y}))\mu_O) \\
&+(1-\mu)\mu[(1-F(\overline{\theta}^S))(1-p(\mathsf{y},\mathsf{y}))\mu_O+F(\underline{\theta}^S)(1-p(\mathsf{n},\mathsf{n}))\mu_O] \\
&+(1-\mu)\mu[F(\overline{\theta}^S)-F(\underline{\theta}^S)]((1-p(\mathsf{y},\mathsf{n}))\mu_O.
\end{aligned} \tag{46}
$$

Noticing that $\lim_{\kappa\to\infty}(F(\overline{\theta}^S) - F(\underline{\theta}^S)) = 0$, and:

$$\lim_{\kappa\to\infty} p(\mathsf{y},\mathsf{y}) = \lim_{\kappa\to\infty} p(\mathsf{n},\mathsf{n}) = \frac{1}{2} + \psi S(\mu - \mu_O) \equiv p,$$

we obtain:

$$\lim_{\kappa\to\infty} Z^S(\kappa) = p(\mu - \mu_O) + \mu_O. \tag{47}$$

Recall that a voter has two representatives under MMD, but only one representative under SMD. In order to ensure to render second-period payoffs comparable across systems, we scale the SMD second period payoff by a factor of two, so that we compare:

$$2\lim_{\kappa\to\infty} Z^S(\kappa) - \lim_{\kappa\to\infty} Z^M(\kappa) = (2(p - p^2) - p^1)(\mu - \mu_O) = \psi S(\mu - \mu_O)^2 > 0. \tag{48}$$